

JEE
Class Companion
Mathematics

For JEE Main and Advanced



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Trigonometric Equation (Phase-II)

Learning Objectives

After studying this chapter, you will be able to:

4-1. learn solution of trigonometric equation

4-2. learn solution of quadratic equation

Solution of Trigonometric Equations

A solution of trigonometric equation is the value of the unknown angle that satisfies the equation.

$$\text{e.g., if } \sin \theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, \dots$$

Thus, the trigonometric equation may have infinite number of solutions (because of their periodic nature) and can be classified as:

1. Principal solution
2. General solution.

Principal Solutions

The solutions of trigonometric equation which lie in the interval $[0, 2\pi)$ are called *principal solutions*.

EXAMPLE 1

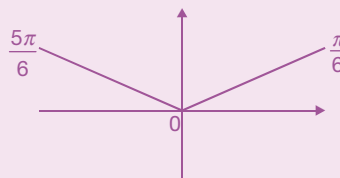
Find the principal solutions of the equation $\sin x = \frac{1}{2}$.

SOLUTION

$\therefore \sin x = \frac{1}{2} \quad \therefore$ there exists two values

i.e., $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ which lie in $[0, 2\pi)$ and whose sine

is $\frac{1}{2}$



\therefore Principal solutions of the equation $\sin x = \frac{1}{2}$ are

$$\frac{\pi}{6}, \frac{5\pi}{6}$$

General Solution

The expression involving an integer 'n' which gives all solutions of a trigonometric equation is called **General Solution**.

1. If $\sin \theta = \sin \alpha \Rightarrow \theta = n\pi + (-1)^n \alpha$ where $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $n \in I$.
2. If $\cos \theta = \cos \alpha \Rightarrow \theta = 2n\pi \pm \alpha$ where $\alpha \in [0, \pi]$, $n \in I$.

3. If $\tan \theta = \tan \alpha \Rightarrow \theta = n\pi + \alpha$ where $\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $n \in I$.
4. If $\sin^2 \theta = \sin^2 \alpha \Rightarrow \theta = n\pi \pm \alpha$.
5. If $\cos^2 \theta = \cos^2 \alpha \Rightarrow \theta = n\pi \pm \alpha$.
6. If $\tan^2 \theta = \tan^2 \alpha \Rightarrow \theta = n\pi \pm \alpha$.

Note

α is called the principal angle.

EXAMPLE 2

Solve: $\sec 2\theta = -\frac{2}{\sqrt{3}}$.

SOLUTION

$$\begin{aligned} \sec 2\theta &= -\frac{2}{\sqrt{3}} \Rightarrow \cos 2\theta = -\frac{\sqrt{3}}{2} \\ \Rightarrow \cos 2\theta &= \cos \frac{5\pi}{6} \Rightarrow 2\theta = 2n\pi \pm \frac{5\pi}{6}, n \in I \\ \Rightarrow \theta &= n\pi \pm \frac{5\pi}{12}, n \in I. \end{aligned}$$

EXAMPLE 3

Solve: $\tan \theta = 2$.

SOLUTION

$$\tan \theta = 2 \quad (1)$$

Let $2 \tan \alpha \Rightarrow \tan \theta = \tan \alpha$
 $\Rightarrow \theta = n\pi + \alpha$, where $\alpha = \tan^{-1}(2)$, $n \in I$.

EXAMPLE 4

Solve: $\cos^2 \theta = \frac{1}{2}$.

SOLUTION

$$\begin{aligned} \cos^2 \theta &= \frac{1}{2} \Rightarrow \cos^2 \theta = \left(\frac{1}{\sqrt{2}}\right)^2 \Rightarrow \cos^2 \theta = \cos^2 \frac{\pi}{4} \\ \Rightarrow \theta &= n\pi \pm \frac{\pi}{4}, n \in I \end{aligned}$$

EXAMPLE 5

Solve: $4 \tan^2 \theta = 3 \sec^2 \theta$.

SOLUTION

$$\therefore 4 \tan^2 \theta = 3 \sec^2 \theta \quad (1)$$

For equation (1) to be defined $\theta \neq (2n+1)\pi$, $n \in I$.

$$\therefore \text{equation (1) can be written as: } \frac{4 \sin^2 \theta}{\cos^2 \theta} = \frac{3}{\cos^2 \theta}$$

$$\therefore \theta \neq (2n+1)\frac{\pi}{2}, n \in I$$

$$\Rightarrow 4 \sin^2 \theta = 3 \quad \therefore \cos^2 \theta \neq 0$$

$$\Rightarrow \sin^2 \theta = \left(\frac{\sqrt{3}}{2}\right)^2 \Rightarrow \sin^2 \theta = \sin^2 \frac{\pi}{3}$$

$$\Rightarrow \theta = n\pi \pm \frac{\pi}{3}, n \in I.$$

Solutions of Equations by Factorising**EXAMPLE 6**

Solve: $(2 \sin x - \cos x)(1 + \cos x) = \sin^2 x$.

SOLUTION

$$\frac{5}{2} (2 \sin x - \cos x)(1 + \cos x) = \sin^2 x$$

$$\begin{aligned} \Rightarrow (2 \sin x - \cos x)(1 + \cos x) - (1 - \cos x) \\ (1 + \cos x) &= 0 \\ \Rightarrow (1 + \cos x)(2 \sin x - 1) &= 0 \\ \Rightarrow 1 + \cos x = 0 \text{ or } 2 \sin x - 1 &= 0 \\ \Rightarrow \cos x = -1 \text{ or } \sin x = \frac{1}{2} \end{aligned}$$

$$\Rightarrow x = (2n + 1)\pi, n \in I \quad \text{or} \quad \sin x = \sin\left(\frac{\pi}{6}\right), n \in I$$

$$\Rightarrow x = n\pi + (-1)^n \frac{\pi}{6}, n \in I$$

\therefore Solution of given equation is

$$(2n + 1)\pi, n \in I \quad \text{or} \quad n\pi + (-1)^n \frac{\pi}{6}, n \in I.$$

EXAMPLE 7

Solve the equation: $\sin^3 x \cos x - \sin x \cos^3 x = \frac{1}{4}$.

SOLUTION

The equation can be written as $4 \sin x \cos x (\sin^2 x - \cos^2 x) = 1$,

$$\Rightarrow -2 \sin 2x \cos 2x = -\sin 4x = 1$$

$$\Rightarrow x = -\frac{3\pi}{2} + k \cdot \frac{\pi}{2} \quad (k = 0, \pm 1, \pm 2, \dots).$$

EXAMPLE 8

Find the general solution of the equation

$$\frac{1 + \sin x + \sin^2 x + \dots + \sin^n x + \dots \infty}{1 - \sin x + \sin^2 x - \sin^3 x + \dots (-1)^n \sin^n x + \dots \infty} = \frac{4}{1 + \tan^2 x}$$

$$\text{where } x \neq k\pi + \frac{\pi}{2}, k \in I.$$

SOLUTION

$$N^r \text{ of LHS} = \frac{1}{1 - \sin x}; \quad D^r \text{ of LHS} = \frac{1}{1 + \sin x}$$

$$\text{hence, } \frac{1 + \sin x}{1 - \sin x} = \frac{4}{\sec^2 x} = 4 \cos^2 x \\ = 4(1 - \sin x)(1 + \sin x)$$

$$\text{hence, } 4(1 - \sin x)^2 = 1 \quad \Rightarrow \quad (1 - \sin x)^2 = \frac{1}{4}$$

$$\Rightarrow (1 - \sin x) = \frac{1}{2} \text{ or } -\frac{1}{2}$$

$$\therefore \sin x = \frac{1}{2} \quad \text{or} \quad \sin x = \frac{3}{2} \text{ (rejected)}$$

$$\therefore \sin x = \sin \frac{\pi}{6} \quad \Rightarrow \quad x = n\pi + (-1)^n \frac{\pi}{6}, n \in I.$$

EXAMPLE 9

Find the general solution of the equation $\sin^3 x(1 + \cot x) + \cos^3 x(1 + \tan x) = \cos 2x$.

SOLUTION

$$\sin^2 x(\cos x + \sin x) + \cos^2 x(\cos x + \sin x) = \cos 2x \\ (\cos x + \sin x)(\cos^2 x + \sin^2 x) = (\cos x + \sin x) \\ (\cos x - \sin x)$$

$$\therefore (\cos x + \sin x)[\cos x - \sin x - 1] = 0$$

$$\therefore \text{either } \cos x + \sin x = 0 \quad (1)$$

$$\text{or } \cos x - \sin x = 1 \quad (2)$$

$$\text{from (1) } \tan x = -1 \text{ or } 1 - \sin 2x = 1 \Rightarrow \sin 2x = 0$$

$$\text{If } \tan x = -1 = \tan\left(-\frac{\pi}{4}\right) \quad \therefore \quad x = n\pi - \frac{\pi}{4}, n \in I.$$

$$\text{If } \sin 2x = 0 \Rightarrow 2x = n\pi \Rightarrow x = \frac{n\pi}{2} \text{ this is to be rejected}$$

because of the $\tan x$ or $\cot x$ will not be defined so

$$x = \left(n\pi - \frac{\pi}{4}\right), n \in I.$$

EXAMPLE 10

Find the solutions of the equation, $\log_{\sqrt{2 \sin x}}(1 + \cos x) = 2$ in the interval $x \in [0, 2\pi]$.

SOLUTION

$$2 \sin^2 x = 1 + \cos x; \quad 2 \cos^2 x + \cos x - 1 = 0$$

$$\Rightarrow \cos x = \frac{1}{2} \quad \text{or} \quad -1 \quad \Rightarrow \quad x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

$$\text{but } x = \pi \text{ and } \frac{5\pi}{3} \text{ are rejected} \quad \Rightarrow \quad x = \frac{\pi}{3}.$$

Solutions of Equations Reducible to Quadratic Equations

EXAMPLE 11

Solve the equation

$$\sin^2 x(\tan x + 1) = 3 \sin x(\cos x - \sin x) + 3.$$

SOLUTION

The given equation makes no sense when $\cos x = 0$; therefore, we can suppose that $\cos x \neq 0$. Noting that the right-hand member of the equation is equal to 3

$\sin x \cos x + 3 \cos^2 x$, and dividing both members by $\cos^2 x$, we obtain $\tan^2 x (\tan x + 1) = 3 (\tan x + 1)$,

$$\Rightarrow (\tan^2 x - 3) (\tan x + 1) = 0$$

$$\Rightarrow x_1 = -\frac{\pi}{4} + k\pi, x_2 = \frac{\pi}{3} + k\pi, x_3 = -\frac{\pi}{3} + k\pi.$$

EXAMPLE 12

Find the general solution set of the equation $\log_{\tan x} (2 + 4 \cos^2 x) = 2$.

SOLUTION

$$2 + 4 \cos^2 x = \tan^2 x$$

$$\Rightarrow 2 + 4 \cos^2 x = \tan^2 x$$

$$\Rightarrow 4 \cos^4 x + 3 \cos^2 x - 1 = 0$$

$$\text{let } \cos^2 x = t \Rightarrow 4t^2 + 3t - 1 = 0$$

$$\Rightarrow (4t - 1)(t + 1) = 0 \Rightarrow t = \frac{\pi}{7} \text{ or } t = -1$$

$$\Rightarrow \cos^2 x = \frac{\pi}{7} \text{ or } \cos^2 x = -1 \text{ (not possible)}$$

$$\Rightarrow \cos^2 x = \cos^2 \frac{\pi}{3} \Rightarrow x = n\pi + \frac{\pi}{3}, n \in I.$$

EXAMPLE 13

The equation $\cos^2 x - \sin x + a = 0$ has roots when

$$x \in \left(0, \frac{\pi}{2}\right) \text{ find 'a'.$$

SOLUTION

$$1 - \sin^2 x - \sin x + a = 0$$

$$\Rightarrow \sin^2 x + \sin x - (a + 1) = 0 \quad (\text{let } \sin x = t)$$

$$\therefore t^2 + t - (a + 1) = 0, t \in (0, 1)$$

$$\Rightarrow t = \frac{-1 \pm \sqrt{1 + 4(a + 1)}}{2}$$

$$\Rightarrow t = \frac{-1 \pm \sqrt{4a + 5}}{2} \text{ (reject -ve sign)}$$

$$\therefore t = \frac{-1 + \sqrt{4a + 5}}{2} \text{ now } 0 < \frac{-1 + \sqrt{4a + 5}}{2} < 1$$

$$\Rightarrow 0 < -1 + \sqrt{4a + 5} < 2 \text{ or } 1 < \sqrt{4a + 5} < 3$$

$$\Rightarrow 1 < 4a + 5 < 9 \Rightarrow -4 < 4a < 4$$

$$\Rightarrow -1 < a < 1 \Rightarrow a \in (-1, 1).$$

EXAMPLE 14

Solve the equation $\sin^8 x + \cos^8 x = \frac{17}{32}$.

SOLUTION

Using the identity $(\sin^2 x + \cos^2 x)^2 = 1$ we get $\sin^4 x +$

$$\cos^4 x = 1 - \frac{1}{2} \sin^2 2x,$$

$$\text{whence, } \sin^8 x + \cos^8 x = \left(1 - \frac{1}{2} \sin^2 2x\right)^2 - \frac{1}{8} \sin^4 2x \\ = \frac{17}{32}$$

$$\Rightarrow 1 - \sin^2 2x + \frac{1}{8} \sin^4 2x = \frac{17}{32}$$

$$\Rightarrow \sin^4 2x - 8 \sin^2 2x + \frac{15}{4} = 0.$$

$$\text{Solving we get } \sin^2 2x = 4 \pm \frac{7}{2}, \sin^2 2x = \frac{1}{2}, 2x = \frac{\pi}{4} + \\ k \frac{\pi}{2}; \text{ whence, } x = \frac{2k + 1}{8} \pi.$$

EXAMPLE 15

Find all solutions of the equation $(\tan^2 x - 1)^{-1} = 1 + \cos 2x$, which satisfy the inequality $2^{x+1} - 8 > 0$.

SOLUTION

Let us reduce the initial trigonometric equation to the

$$\text{form } (1 + \cos 2x) \left(1 + \frac{1}{2 \cos 2x}\right) = 0.$$

The following values of x are solutions of this equation

$$x = -\frac{\pi}{2} + \pi n, x = \pm \frac{\pi}{3} + \pi k, n, k \in \mathbb{Z}.$$

By the hypothesis, we must choose those values of x which satisfy the inequalities

$$2^{x+1} - 8 > 0, \cos x \neq 0. \text{ The values we need are } x = \pm \frac{\pi}{3} + \pi n, \\ n \in \mathbb{N}.$$

EXAMPLE 16

Determine all the values of a for which the equation $\sin^4 x - 2 \cos^2 x + a^2 = 0$ is solvable. Find the solutions.

SOLUTION

$$\text{Applying the formula } \sin^4 x = \left(\frac{1 - \cos 2x}{2}\right)^2, \cos^2 x = \\ \frac{1 + \cos 2x}{2} \text{ and putting } \cos 2x = t$$

we rewrite the given equation in the form

$$t^2 - 6t + 4a^2 - 3 = 0 \quad (1)$$

The original equation has solutions for a given value of a if and only if, for this value of a , the roots t_1 and t_2 of the equation (1) are real and at least one of these roots does not exceed unity in its absolute value. Solving equation (1), we find $t_1 = 3 - 2\sqrt{3-a^2}$, $t_2 = 3 + 2\sqrt{3-a^2}$.

Hence, the roots of equation (1) are real if

$$|a| \leq \sqrt{3} \quad (2)$$

If condition (2) is fulfilled, then $t_2 > 1$ and, therefore, this root can be discarded. Thus, the problem is reduced to finding the values of a satisfying condition (2), for which $|t_1| \leq 1$, i.e.,

$$-1 \leq 3 - 2\sqrt{3-a^2} \leq 1. \quad (3)$$

From (3) we find $-4 \leq -2\sqrt{3-a^2} \leq -2$,

$$\Rightarrow 2 \geq \sqrt{3-a^2} \geq 1. \quad (4)$$

Since the inequality $2 \geq \sqrt{3-a^2}$ is fulfilled for $|a| \leq \sqrt{3}$, the system of inequalities (4) is reduced to the inequality $\sqrt{3-a^2} \geq 1$, $\Rightarrow |a| \leq \sqrt{2}$.

Thus, the original equation is solvable if $|a| \leq \sqrt{2}$, and its solutions are

$$x = \pm \frac{1}{2} \arccos(3 - 2\sqrt{3-a^2}) + k\pi.$$

Solving Equations by Introducing an Auxiliary Argument

EXAMPLE 17

Solve: $\sin x + \cos x = \sqrt{2}$.

SOLUTION

$$\therefore \sin x + \cos x = \sqrt{2} \quad (1)$$

Here, $a = 1$, $b = 1$

\therefore divide both sides of equation (1) by $\sqrt{2}$,

$$\text{we get } \sin x \cdot \frac{1}{\sqrt{2}} + \cos x \cdot \frac{1}{\sqrt{2}} = 1$$

$$\Rightarrow \sin x \cdot \sin \frac{\pi}{4} + \cos x \cdot \cos \frac{\pi}{4} = 1$$

$$\Rightarrow \cos \left(x - \frac{\pi}{4} \right) = 1$$

$$\Rightarrow x - \frac{\pi}{4} = 2n\pi, n \in I \quad \Rightarrow \quad x = 2n\pi + \frac{\pi}{4}, n \in I$$

\therefore Solution of given equation is $2n\pi + \frac{\pi}{4}, n \in I$.

EXAMPLE 18

Solve the equation $\cos 7x - \sin 5x = \sqrt{3} (\cos 5x - \sin 7x)$.

SOLUTION

Rewrite the equation in the form $\frac{1}{2} \cos 7x + \frac{\sqrt{3}}{2} \sin 7x = \frac{\sqrt{3}}{2} \cos 5x + \frac{1}{2} \sin 5x$

or

$$\sin \frac{\pi}{6} \cos 7x + \cos \frac{\pi}{6} \sin 7x = \sin \frac{\pi}{3} \cos 5x + \cos \frac{\pi}{3} \sin 5x,$$

$$\text{i.e., } \sin \left(\frac{\pi}{6} + 7x \right) = \sin \left(\frac{\pi}{3} + 5x \right).$$

But $\sin \alpha = \sin \beta$ if and only if either $\alpha - \beta = 2k\pi$ or $\alpha + \beta = (2m+1)\pi$ ($k, m = 0, \pm 1, \pm 2, \dots$).

$$\text{Hence, } \frac{\pi}{6} + 7x - \frac{\pi}{6} - 5x = 2k\pi \text{ or } \frac{\pi}{6} + 7x - \frac{\pi}{3} - 5x = (2m+1)\pi.$$

Thus, the roots for the equation are

$$\left. \begin{aligned} x &= \frac{\pi}{12}(12k+1) \\ x &= \frac{\pi}{24}(4m+1) \end{aligned} \right\} (k, m = 0, \pm 1, \pm 2, \dots).$$

EXAMPLE 19

Solve the equation $2 \sin 17x + \sqrt{3} \cos 5x + \sin 5x = 0$.

SOLUTION

Dividing both sides of the equation by 2, we reduce it

to the form $\sin 17x + \sin\left(5x + \frac{\pi}{3}\right) = 0$,

whence, we obtain $2 \sin\left(11x + \frac{\pi}{6}\right) \cos\left(6x - \frac{\pi}{6}\right) = 0$

$$\Rightarrow x_1 = -\frac{\pi}{66} + \frac{k\pi}{11}, x_2 = \frac{\pi}{36} + \frac{(2k+1)\pi}{12}.$$

EXAMPLE 20

Solve the equation $\sin^3 x + \cos^3 x = 1 - \frac{1}{2} \sin 2x$.

SOLUTION

Using the formula for the sum of cubes of two members we transform the Left-hand Side (L.H.S) of the equation in the following way: $(\sin x + \cos x)(1 - \sin x \cos x)$

$$= \left(1 - \frac{1}{2} \sin 2x\right) (\sin x + \cos x).$$

Hence, the original equation takes the form

$$\left(1 - \frac{1}{2} \sin 2x\right) (\sin x + \cos x - 1) = 0.$$

The expression in the first brackets is different from zero for all x . Therefore, it is sufficient to consider the equation $\sin x + \cos x - 1 = 0$. The latter is reduced to the form

$$\sin\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \Rightarrow x_1 = 2\pi k, x_2 = \frac{\pi}{2} + 2\pi k.$$

Solving Equations by Transforming a Sum of Trigonometric Functions Into a Product

EXAMPLE 21

Solve: $\cos 3x + \sin 2x - \sin 4x = 0$.

SOLUTION

$$\cos 3x + \sin 2x - \sin 4x = 0$$

$$\Rightarrow \cos 3x + 2 \cos 3x \cdot \sin(-x) = 0$$

$$\Rightarrow \cos 3x - 2 \cos 3x \cdot \sin x = 0$$

$$\Rightarrow \cos 3x (1 - 2 \sin x) = 0$$

$$\Rightarrow \cos 3x = 0 \quad \text{or} \quad 1 - 2 \sin x = 0$$

$$\Rightarrow 3x = (2n+1) \frac{\pi}{2}, n \in I \quad \text{or} \quad \sin x = \frac{1}{2}$$

$$\Rightarrow x = (2n+1) \frac{\pi}{6}, n \in I \quad \text{or} \quad x = n\pi + (-1)^n \frac{\pi}{6}, n \in I$$

\therefore Solution of given equation is

$$(2n+1) \frac{\pi}{6}, n \in I \quad \text{or} \quad n\pi + (-1)^n \frac{\pi}{6}, n \in I.$$

Solving Equations by Transforming a Product of Trigonometric Functions Into a Sum

EXAMPLE 22

Solve: $\sin 5x \cdot \cos 3x = \sin 6x \cdot \cos 2x$.

SOLUTION

$$\therefore \sin 5x \cdot \cos 3x = \sin 6x \cdot \cos 2x$$

$$\Rightarrow 2 \sin 5x \cdot \cos 3x = 2 \sin 6x \cdot \cos 2x$$

$$\Rightarrow \sin 8x + \sin 2x = \sin 8x + \sin 4x$$

$$\Rightarrow \sin 4x - \sin 2x = 0$$

$$\Rightarrow 2 \sin 2x \cdot \cos 2x - \sin 2x = 0$$

$$\Rightarrow \sin 2x (2 \cos 2x - 1) = 0$$

$$\Rightarrow \sin 2x = 0 \quad \text{or} \quad 2 \cos 2x - 1 = 0$$

$$\Rightarrow 2x = n\pi, n \in I \quad \text{or} \quad \cos 2x = \frac{1}{2}$$

$$\Rightarrow x = \frac{n\pi}{2}, n \in I \quad \text{or} \quad 2x = 2n\pi \pm \frac{\pi}{3}, n \in I$$

$$\Rightarrow x = n\pi \pm \frac{\pi}{6}, n \in I$$

\therefore Solution of given equation is

$$\frac{n\pi}{2}, n \in I \quad \text{or} \quad n\pi \pm \frac{\pi}{6}, n \in I.$$

EXAMPLE 23

Solve the equation $\cot x - 2 \sin 2x = 1$.

SOLUTION

1. The equation becomes senseless for $x = k\pi$. For all the other values of x it is equivalent to the equation $\cos x - \sin x = 2 \sin 2x \cdot \sin x$ we obtain $\cos x - \sin x = \cos x - \cos 3x$, $\sin x = \cos 3x$,

whence, $\sin x = \sin\left(\frac{\pi}{2} - 3x\right)$. Consequently,

$$2 \sin\left(2x - \frac{\pi}{4}\right) \cos\left(x - \frac{\pi}{4}\right) = 0$$

$$\Rightarrow x_1 = \frac{\pi}{8} + \frac{k\pi}{2}, x_2 = \frac{4\pi}{4} + k\pi.$$

2. Putting $\tan x = t$, we get the equation $t^3 + 3t^2 + t - 1 = 0$.

Factoring the left member, we obtain

$$(t + 1)(t + 1 - \sqrt{2})(t + 1 + \sqrt{2}) = 0.$$

hence, $(\tan x_1) = 1$, $(\tan x_2) = \sqrt{2} - 1$, $(\tan x_3) = -1 - \sqrt{2}$.

$$\Rightarrow x_1 = \frac{3\pi}{4} + k\pi; x_2 = \arctan(\sqrt{2} - 1) + k\pi,$$

$$x_3 = -\arctan(1 + \sqrt{2}) + k\pi.$$

Solving Equations by a Change of Variable

- Equations of the form $P(\sin x \pm \cos x, \sin x \cdot \cos x) = 0$, where $P(y, z)$ is a polynomial, can be solved by the change $\cos x \pm \sin x = t \Rightarrow 1 \pm 2 \sin x \cdot \cos x = t^2$.
- Equations of the form $a \cdot \sin x + b \cdot \cos x + d = 0$, where a, b and d are real numbers and $a, b \neq 0$ can be solved by changing $\sin x$ and $\cos x$ into their corresponding tangent of half the angle.
- Many equations can be solved by introducing a new variable. e.g. the equation $\sin^4 2x + \cos^4 2x = \sin 2x \cdot \cos 2x$ changes to $2(y + 1)\left(y - \frac{1}{2}\right) = 0$ by substituting, $\sin 2x \cdot \cos 2x = y$.

EXAMPLE 24

Solve $\sin x + \cos x = 1 \sin x \cdot \cos x$.

SOLUTION

$$\therefore \sin x + \cos x = 1 \sin x \cdot \cos x \quad (1)$$

Let $\sin x + \cos x = t$

$$\Rightarrow \sin^2 x + \cos^2 x + 2 \sin x \cdot \cos x = t^2$$

$$\Rightarrow \sin x \cdot \cos x = \frac{t^2 - 1}{2}$$

Now put $\sin x + \cos x = t$ and $\sin x \cdot \cos x = \frac{t^2 - 1}{2}$ in

$$(1), \text{ we get } t = 1 + \frac{t^2 - 1}{2}$$

$$\Rightarrow t^2 - 2t + 1 = 0 \Rightarrow t = 1 \quad (\because t = \sin x + \cos x)$$

$$\Rightarrow \sin x + \cos x = 1 \quad (2)$$

divide both sides of equation (2) by $\sqrt{2}$, we get

$$\Rightarrow \sin x \cdot \frac{1}{\sqrt{2}} + \cos x \cdot \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \cos \frac{\pi}{4}$$

$$\Rightarrow x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$$

(a) if we take positive sign, we get $x = 2n\pi + \frac{\pi}{4}$, $n \in I$.

(b) if we take negative sign, we get $x = 2n\pi$, $n \in I$.

EXAMPLE 25

Solve the equation $\sin 2x - 12(\sin x - \cos x) + 12 = 0$.

SOLUTION

Putting $\sin x - \cos x = t$ and using the identity $(\sin x - \cos x)^2 = 1 - 2 \sin x \cos x$, we rewrite the original equation in the form $t^2 + 12t - 13 = 0$.

This equation has the roots $t_1 = -13$ and $t_2 = 1$.

But $t = \sin x - \cos x = \sqrt{2} \sin\left(x - \frac{\pi}{4}\right)$, and thus, $|t| \leq \sqrt{2}$. Consequently, the root $t_1 = -13$ must be discarded. Therefore, the original equation is reduced to the equation $\sin\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$.

$$\Rightarrow x_1 = \pi + 2k\pi, x_2 = \frac{\pi}{2} + 2k\pi.$$

EXAMPLE 26

Solve the equation $1 + 2 \operatorname{cosec} x = -\frac{\sec^2 x}{2}$.

SOLUTION

Transform the given equation to the form $2 \cos^2 \frac{x}{2} (2 + \sin x) + \sin x = 0$.

Using the formula $2 \cos^2 \frac{x}{2} = 1 + \cos x$ and opening the brackets, we obtain

$$2 + 2(\sin x + \cos x) + \sin x \cdot \cos x = 0 \quad (1)$$

By the substitution $\sin x + \cos x = t$ equation (1), is reduced to the quadratic equation $t^2 + 4t + 3 = 0$ whose roots are $t_1 = -1$ and $t_2 = -3$. Since $|\sin x + \cos x| \leq \sqrt{2}$, the original equation can only be satisfied by the roots of the equation

$$\sin x + \cos x = -1 \quad (2)$$

Solving equation (2), we obtain $x_1 = -\frac{\pi}{2} + 2k\pi$ and $x_2 = (2k+1)\pi$.

Here, x_2 should be discarded because $\sin x_2 = 0$, and therefore, the original equation makes no sense for

$$x = x_2 \Rightarrow x = -\frac{\pi}{2} + 2k\pi.$$

EXAMPLE 27

Solve the equation $\sin x + \cos x - 2\sqrt{2} \sin x \cos x = 0$.

SOLUTION

Designating $\sin x + \cos x = t$ and using the equation $\sin x \cos x = \frac{(t^2 - 1)}{2}$, we reduce the equation to a new equation with respect to t : $\sqrt{2}t^2 - t - \sqrt{2} = 0$.

The numbers $t_1 = \sqrt{2}$, $t_2 = -\frac{1}{\sqrt{2}}$ are roots of this quadratic equation.

Thus, the solution of the initial equation reduces to the solution of the trigonometric equations:

$$\sin x + \cos x = \sqrt{2}, \sin x + \cos x = -\frac{1}{\sqrt{2}}.$$

Multiplying both sides of these equations by the number $\frac{1}{\sqrt{2}}$, we reduce them to two simpler equations:

$$\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = 1$$

$$\Rightarrow \sin x \cos \frac{\pi}{4} + \sin \frac{\pi}{4} \cos x = 1 \Rightarrow \sin\left(x + \frac{\pi}{4}\right) = 1.$$

$$\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = -\frac{1}{2} \Rightarrow \sin\left(x + \frac{\pi}{4}\right) = -\frac{1}{2}.$$

The solutions of the equations $\sin\left(x + \frac{\pi}{4}\right) = 1$ and $\sin\left(x + \frac{\pi}{4}\right) = -\frac{1}{2}$ are

$$x = \frac{\pi}{4} + 2\pi k, k \in \mathbb{Z}; x = (-1)^{n+1} \frac{\pi}{6} - \frac{\pi}{4} + \pi n, n \in \mathbb{Z}.$$

EXAMPLE 28

Solve the equation

$$\frac{1}{2}(\sin^4 x + \cos^4 x) = \sin^2 x \cos^2 x + \sin x \cos x.$$

SOLUTION

We obtain the equation $\sin^2 2x + \sin 2x - 1 = 0$.

Solving it, we get $\sin 2x = \frac{\sqrt{5}-1}{2}$

$$\Rightarrow x = (-1)^k \frac{1}{2} \arcsin \frac{\sqrt{5}-1}{2} + \frac{k\pi}{2}.$$

EXAMPLE 29Solve: $3 \cos x + 4 \sin x = 5$.**SOLUTION**

$$\therefore 3 \cos x + 4 \sin x = 5$$

$$\therefore \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \text{ and } \sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \quad (1)$$

\therefore equation (1) becomes

$$\Rightarrow 3 \left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right) + 4 \left(\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right) = 5 \quad (2)$$

Let $\tan \frac{x}{2} = t$

$$\therefore \text{equation (1) becomes } 3 \left(\frac{1 - t^2}{1 + t^2} \right) + 4 \left(\frac{2t}{1 + t^2} \right) = 5$$

$$\Rightarrow 4t^2 - 4t + 1 = 0$$

$$\Rightarrow (2t - 1)^2 = 0 \Rightarrow t = \frac{1}{2} \quad (\because t = \tan \frac{x}{2})$$

$$\Rightarrow \tan \frac{x}{2} = \frac{1}{2} \Rightarrow \tan \frac{x}{2} = \tan \alpha, \text{ where } \tan \alpha = \frac{1}{2}$$

$$\Rightarrow \tan \frac{x}{2} = n\pi + \alpha \Rightarrow x = 2n\pi + 2\alpha$$

$$\text{where } \alpha = \tan^{-1} \left(\frac{1}{2} \right), n \in I.$$

**CHECK YOUR UNDERSTANDING-I**

1. Find the number of solutions of the trigonometric equation $4 \cos^2 x + \tan^2 x + \cot^2 x + \sec^2 x = 6$ in $[0, 2\pi]$.
2. Consider the trigonometric equation $\tan x(\sin 2x + 1) = \sin x(2 + \tan x)$. Find the number of solutions of the equation in $(0, 4\pi)$.
3. Find real values of x for which, $27^{\cos 2x} \cdot 81^{\sin 2x}$ is minimum. Also find this minimum value.
4. Solve the equation: $2 \sin x = 3x^2 + 2x + 3$.
5. Solve: $2 + 7 \tan^2 \theta = 3.25 \sec^2 \theta$ ($0^\circ < \theta < 360^\circ$).

Solving Equations with the Use of the Boundness of the Functions $\sin x$ and $\cos x$

EXAMPLE 30Solve the equation $\frac{1 - \tan x}{1 + \tan x} = 1 + \sin 2x$.**SOLUTION**

The equation makes no sense for $x = \frac{\pi}{2} + k\pi$ and for $x = -\frac{\pi}{4} + k\pi$. For all the other values of x it is equivalent to the equation

$$\frac{\cos x - \sin x}{\cos x + \sin x} = 1 + \sin 2x.$$

After simple transformations we obtain $\sin x(3 + \sin 2x + \cos 2x) = 0$.

It is obvious that the equation $\sin 2x + \cos 2x + 3 = 0$ has no solution, and therefore, the original equation is reduced to the equation $\sin x = 0 \Rightarrow x = k\pi$

EXAMPLE 31Solve the equation $(\sin x + \cos x)\sqrt{2} = \tan x + \cot x$.**SOLUTION**

Let us transform the equation to the form

$$\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = \frac{1}{2 \sin x \cos x} \text{ or } \sin$$

$$\left(x + \frac{\pi}{4} \right) = \frac{1}{\sin 2x},$$

$$\text{i.e., } \sin \left(x + \frac{\pi}{4} \right) \sin 2x = 1. \quad (1)$$

We have $|\sin \alpha| \leq 1$, and therefore (1) holds if either \sin

$$\left(x + \frac{\pi}{4} \right) = 1 \text{ and } \sin 2x = -1 \text{ or } \sin \left(x + \frac{\pi}{4} \right) = 1 \text{ and } \sin 2x = 1.$$

But the first two equations have no roots in common while the second two equations have the common

roots $x = \frac{\pi}{4} + 2k\pi$. Consequently, the roots of the given equation are $x = \frac{\pi}{4} + 2k\pi$.

EXAMPLE 32

Solve the equation $\sin^{2n-1} x + 2 \cos^{2n-1} x = 2$, where $n \in \mathbb{N}$.

SOLUTION

Obviously no solution is possible if $\frac{\pi}{2} < x < 2\pi$ as $\text{LHS} < 2$.

If $0 < x < \frac{\pi}{2}$, then $\text{LHS} = \sin^{2n-1} x + 2 \cos^{2n-1} x < \sin^2 x + 2 \cos^2 x = 1 + \cos^2 x < 2$ when $n \in \mathbb{N} - \{1\}$.

Obviously, a solution exists only when $x = 0$

\Rightarrow The general solution is $x = 2m\pi, m \in \mathbb{I}$.

When $n = 1$, $\sin x + 2 \cos x = 2 \sin \frac{x}{2} \left(2 \sin \frac{x}{2} - \cos \frac{x}{2} \right) = 0$

\Rightarrow either $x = 2k_1\pi$ or $x = 2k_2\pi + 2 \tan^{-1} \frac{1}{2}, k_1, k_2 \in \mathbb{I}$.

EXAMPLE 33

Solve the equation $\cos^2 \left[\frac{\pi}{4} (\sin x + \sqrt{2} \cos^2 x) \right] - \tan^2 \left(x + \frac{\pi}{4} \tan^2 x \right) = 1$.

SOLUTION

$$\cos^2 \left[\frac{\pi}{4} (\sin x + \sqrt{2} \cos^2 x) \right] - \tan^2 \left(x + \frac{\pi}{4} \tan^2 x \right) = 1$$

since square of the cosine of any argument doesn't exceed 1, the given equation holds true if and only if we have, simultaneously

$$\cos^2 \left[\frac{\pi}{4} (\sin x + \sqrt{2} \cos^2 x) \right] = 1 \quad (1)$$

and

$$\tan \left(x + \frac{\pi}{4} \tan^2 x \right) = 0 \quad (2)$$

from (1), $\sin x + \sqrt{2} \cos^2 x = 4k \quad (3)$

$$\forall k \in \mathbb{I} \text{ but } |\sin x + \sqrt{2} \cos^2 x| \leq |\sin x| + \sqrt{2} |\cos^2 x| \leq 1 + \sqrt{2} < 4$$

so, equation (3) has no solution for $k \neq 0$ for $k = 0$

$$\sin x + \sqrt{2} \cos^2 x = 0 \text{ or, } \sqrt{2} \sin^2 x - \sin x - \sqrt{2} = 0 \text{ or,}$$

$$\sin x = \frac{-1 \pm \sqrt{2}}{\sqrt{2}}$$

but $\sin x = \sqrt{2}$ is not possible. So only solution to the equation (1) is

$$x_1 = \frac{-\pi}{4} + 2n\pi, x_2 = \frac{5\pi}{4} + 2n\pi, n = 0, \pm 1, \pm 2 \dots$$

for $x_1 = \frac{-\pi}{4} + 2n\pi$, equation (2) becomes an identity

but $x_2 = \frac{5\pi}{4} + 2n\pi$ doesn't satisfy equation (2)

so, solution to the original equation $x = \frac{-\pi}{4} + 2n\pi \forall n \in \mathbb{I}$.

EXAMPLE 34

Find the general solution of the equation, $\sin 3x + \cos 4x - 4 \sin 7x = \cos 10x + \sin 17x$.

SOLUTION

$$(\sin 17x - \sin 3x) + \cos 10x - \cos 4x + 4 \sin 7x = 0$$

$$\Rightarrow 2 \cos 10x \sin 7x + 2 \sin 7x \sin 3x + 4 \sin 7x = 0$$

$$\Rightarrow \sin 7x (\cos 10x - \sin 3x + 2) = 0$$

$$\text{Hence, } \sin 7x = 0$$

$$\Rightarrow x = \frac{n\pi}{7}, n \in \mathbb{I} \text{ or } \cos 10x - \sin 3x + 2 = 0$$

$$\Rightarrow \cos 10x = -1 \text{ and } \sin 3x = 1 \text{ given } x = (4n + 1) \frac{\pi}{6}$$

$$\text{i.e., } x = -\frac{3\pi^*}{6}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{9\pi^*}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{21\pi^*}{6}, \dots,$$

$$\frac{33\pi}{6} \dots$$

Those starred also satisfy $\cos 10x = -1$, the general term of which is

$$x = 3(4k - 1) \frac{\pi}{6} \quad k \in \mathbb{I}. \text{ Hence, } x = \frac{n\pi}{7} \text{ or } 3(4k - 1) \frac{\pi}{6}$$

where $n, k \in \mathbb{I}$.

Simultaneous Equations

EXAMPLE 35

Solve the system of equations form

$$\left. \begin{aligned} \sin x &= \csc x + \sin y, \\ \cos x &= \sec x + \cos y. \end{aligned} \right\}$$

SOLUTION

Transform the system to the

$$\left. \begin{aligned} \sin^2 x &= 1 + \sin x \sin y, \\ \cos^2 x &= 1 + \cos x \cos y. \end{aligned} \right\} \quad (1)$$

Adding together the equations of system (1) and subtracting the first equation from the second we obtain the system

$$\left. \begin{aligned} \cos 2x - \cos(x+y) &= 0, \\ 1 + \cos(x+y) &= 0. \end{aligned} \right\} \quad (2)$$

The first equation of system (2) can be rewritten as

$$\cos 2x - \cos(x+y) = 2 \sin\left(\frac{3x+y}{2}\right) \sin(y-x) = 0.$$

If $\sin(x-y) = 0$, then $x-y = k\pi$. But from the second equation of system (2) we find

$$\cos(x-y) = -1, \quad x-y = (2n+1)\pi.$$

Consequently, in this case we have an infinitude of solutions: $x-y = (2n+1)\pi$.

If $\sin\left(\frac{3x+y}{2}\right) = 0$, then $3x+y = 2k\pi$. But $x-y =$

$$(2n+1)\pi$$

$$\Rightarrow \quad x = \frac{2k+2n+1}{4}\pi, \quad y = \frac{2k-6n-3}{4}\pi.$$

EXAMPLE 36

Solve the system of equations $\sin x \sin y = \frac{\sqrt{3}}{4}$, $\cos x \cos y = \frac{\sqrt{3}}{4}$.

SOLUTION

Adding up the equations of the system, we arrive at an equation

$$\sin x \sin y + \cos x \cos y = \frac{\sqrt{3}}{2} \Leftrightarrow \cos(x-y) = \frac{\sqrt{3}}{2}.$$

Subtracting the first equation of the system from the second, we arrive at an equation

$$\cos x \cos y - \sin x \sin y = 0 \Leftrightarrow \cos(x+y) = 0,$$

Thus, the initial system is equivalent to the system

$$\cos(x-y) = \frac{\sqrt{3}}{2}, \quad x-y = \pm \frac{\pi}{6} + 2\pi n, \quad \Leftrightarrow n, k \in \mathbb{Z}, \quad \cos(x+y) = 0,$$

$$x+y = \frac{\pi}{2} + \pi k, \quad \text{whence, } x = \frac{\pi}{3} + \frac{\pi}{2}(2n+k),$$

$$x = \frac{\pi}{6} + \frac{\pi}{2}(2n+k),$$

$$y = \frac{\pi}{6} + \frac{\pi}{2}(k-2n), \quad y = \frac{\pi}{3} + \frac{\pi}{2}(k-2n).$$

$$\Rightarrow \quad \frac{\pi}{3} + \frac{\pi}{2}(2n-k), \quad \frac{\pi}{6} + \frac{\pi}{2}(k-2n); \quad \frac{\pi}{6} + \frac{\pi}{2}(2n+k),$$

$$\frac{\pi}{3} + \frac{\pi}{2}(k-2n) \quad (k, n \in \mathbb{Z}).$$

Miscellaneous Questions

EXAMPLE 37

Solve the equation: $2 \cot 2x - 3 \cot 3x = \tan 2x$.

SOLUTION

The given equation can be rewritten in the

$$\text{form } 3 \left(\frac{\cos 2x}{\sin 2x} - \frac{\cos 3x}{\sin 3x} \right) = \frac{\sin 2x}{\cos 2x} + \frac{\cos 2x}{\sin 2x} \text{ or}$$

$$\frac{3 \sin x}{\sin 2x \sin 3x} = \frac{1}{\sin 2x \cos 2x}.$$

Notes

This equation has sense if the condition $\sin 2x \neq 0$, $\sin 3x \neq 0$, $\cos 2x \neq 0$ holds. For the values of x satisfying this condition, we have $3 \sin x \cos 2x = \sin 3x$. Transforming the last equation, we obtain $\sin x (3 - 4 \sin^2 x - 3 \cos 2x) = 0$ and thus, arrive at the equation $2 \sin^3 x = 0$, which is equivalent to the equation $\sin x = 0$. Hence, due to the above note, the original equation has no solutions.

EXAMPLE 38

Solve the equation

$$\tan\left(x - \frac{\pi}{4}\right) \tan x \tan\left(x + \frac{\pi}{4}\right) = \frac{4 \cos^2 x}{\tan \frac{x}{2} - \cot \frac{x}{2}}$$

SOLUTION

The right-hand side (R.H.S.) of the equation is not determined for $x = k\pi$ and $x = \frac{\pi}{2} + m\pi$, because for $x = \frac{\pi}{2}$ the function $\cot \frac{x}{2}$ is not defined, for $x = \left(\frac{2}{\pi} + 1\right)\pi$ the function $\tan \frac{x}{2}$ is not defined and for $x = \frac{\pi}{2} + m\pi$ the denominator of the right member of the right member vanishes. For $x \neq k\pi$ we have

$$\tan \frac{x}{2} - \cot \frac{x}{2} = \frac{\sin^2 \frac{x}{2} - \cos^2 \frac{x}{2}}{\sin \frac{x}{2} \cos \frac{x}{2}} = -\frac{\cos x}{\sin x}$$

Hence, for $x \neq k\pi$ and $x \neq \frac{\pi}{2} + m\pi$ (where k and m are arbitrary integers) the right member of the equation is equal to $-2 \sin x \cos x$.

The left member of the equation has no sense for $x = \frac{\pi}{2} + k\pi$ and $x = \frac{\pi}{4} + l \cdot \frac{\pi}{2}$ ($l = 0, \pm 1, \pm 2, \dots$), and for all the other values of x it is equal to $-\tan x$ because

$$\begin{aligned} \tan\left(x - \frac{\pi}{4}\right) \tan\left(x + \frac{\pi}{4}\right) &= \tan\left(x - \frac{\pi}{4}\right) \cot\left[\frac{\pi}{2} - \left(x + \frac{\pi}{4}\right)\right] \\ &= -\tan\left(x - \frac{\pi}{4}\right) \cot\left(x - \frac{\pi}{4}\right) = -1 \end{aligned}$$

Thus, if $x \neq k\pi$, $x \neq \frac{\pi}{2} + m\pi$ and $x \neq \frac{\pi}{4} + l \cdot \frac{\pi}{2}$, then the original equation is reduced to the form

$$\tan x = 2 \sin x \cos x.$$

This equation has the roots $x = k\pi$ and $x = \frac{\pi}{4} + l \cdot \frac{\pi}{2}$.

It follows that the original equation has no roots.

EXAMPLE 39

Solve the equation $(1 + k) \cos x \cos(2x - \alpha) = (1 + k \cos 2x) \cos(x - \alpha)$.

SOLUTION

Let us rewrite the given equation in the form

$$(1 + k) \cos x \cos(2x - \alpha) = \cos(x - \alpha) + k \cos 2x \cos(x - \alpha). \quad (1)$$

We have $\cos x \cos(2x - \alpha) = \frac{1}{2} [\cos(3x - \alpha) + \cos(x - \alpha)]$

and $\cos(x - \alpha) \cos 2x = \frac{1}{2} [\cos(3x - \alpha) + \cos(x + \alpha)]$,

and therefore, equation (1) turns into $k[\cos(x - \alpha) - \cos(x + \alpha)] = \cos(x - \alpha) - \cos(3x - \alpha)$,

that is $k \sin x \sin \alpha = \sin(2x - \alpha) \sin x. \quad (2)$

Equation (2) is equivalent to the following two equations:

- (a) $\sin x = 0; x = l\pi$ and
- (b) $\sin(2x - \alpha) = k \sin \alpha.$

Thus, $x = \frac{\alpha}{2} + (-1)^n \cdot \frac{1}{2} \arcsin(k \sin \alpha) + \frac{\pi}{2} n$.

For the last expression to make sense, k and α must satisfy the condition $|k \sin \alpha| \leq 1$.

EXAMPLE 40

Solve the equation $(\cos x - \sin x) \left(2 \tan x + \frac{1}{\cos x}\right) + 2 = 0$.

SOLUTION

We designate $t = \tan \frac{x}{2}$ and, using the formulas of the universal trigonometric substitution, write the equation in the form $\frac{3t^4 + 6t^3 + 8t^2 - 2t - 3}{(t^2 + 1)(1 - t^2)} = 0$, its roots

are $t_1 = \frac{1}{\sqrt{3}}, t_2 = -\frac{1}{\sqrt{3}}$. Thus, the solution of the equation reduces that of two elementary equations

$$\tan \frac{x}{2} = \frac{1}{\sqrt{3}}, \tan \frac{x}{2} = -\frac{1}{\sqrt{3}} \quad (1)$$

Verification shows that the numbers πn which are roots of the equation $\cos \frac{\pi}{2} = 0$, are not the roots of the given equation, and consequently, all solutions of the initial equation can be found as solutions of equation (1)

$$\Rightarrow x = \pm \frac{\pi}{3} + 2\pi k, \quad (k \in \mathbb{Z}).$$

EXAMPLE 41

Solve the equation, $5 \sin x + \frac{5}{2 \sin x} - 5 = 2 \sin^2 x + \frac{1}{2 \sin^2 x}$ if $x \in (0, \pi)$.

SOLUTION

$$\begin{aligned} 5 \left(\sin x + \frac{1}{2 \sin x} \right) - 5 &= 2 \left(\sin^2 x + \frac{1}{4 \sin^2 x} \right) \\ &= 2 \left(\left(\sin x + \frac{1}{2 \sin x} \right)^2 - 1 \right) \end{aligned}$$

$$\text{Let } \sin x + \frac{1}{2 \sin x} = t \Rightarrow 5t - 5 = 2(t^2 - 1)$$

$$\Rightarrow 2t^2 - 5t + 3 = 0 \Rightarrow (2t - 3)(t - 1) = 0$$

$$\Rightarrow t = 1 \text{ or } t = \frac{3}{2}$$

If $t = 1$, $2 \sin^2 x - 2 \sin x + 1 = 0 \Rightarrow D < 0$ hence, no solution

$$\text{If } t = \frac{3}{2}, 2 \sin^2 x - 3 \sin x + 1 = 0$$

$$\Rightarrow \sin x = 1 \text{ or } \sin x = \frac{1}{2}$$

$$\therefore x = \frac{\pi}{2} \text{ or } \frac{\pi}{6}, \frac{5\pi}{6} \Rightarrow x \in \left\{ \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6} \right\}$$

EXAMPLE 42

Let A and B be acute positive angles satisfying the equalities $3 \sin^2 A + 2 \sin^2 B = 1$; $3 \sin 2A - 2 \sin 2B = 0$.

Prove that $A + 2B = \frac{\pi}{2}$.

SOLUTION

From the given relations, we get $\sin 2B = \frac{3}{2} \sin 2A$,

$3 \sin^2 A = 1 - 2 \sin^2 B = \cos 2B$, hence, $\cos(A + 2B) = \cos A \cos 2B - \sin A \sin 2B = \cos A \cdot 3 \sin^2 A - \frac{3}{2} \sin A \sin 2A = 0$.

**CHECK YOUR UNDERSTANDING-II**

- Solve the following system of equation for x and y
 $5^{(\cos^2 x - 3 \sec^2 y)} = 1$ and $2^{(2 \operatorname{cosec} x + \sqrt{3} |\sec y|)} = 64$.
- Find the general solution of the equation, $2 + \tan x \cdot \cot \frac{x}{2} + \cot x \cdot \tan \frac{x}{2} = 0$.
- Solve for x , the equation $\sqrt{13 - 8 \tan x} = 6 \tan x - 3$, where $-2\pi < x < 2\pi$.
- Find all values of θ between 0° & 180° satisfying the equation;
 $\cos 6\theta + \cos 4\theta + \cos 2\theta + 1 = 0$.
- Find the general solution of the following equation:
 $2(\sin x - \cos 2x) - \sin 2x(1 + 2 \sin x) + 2 \cos x = 0$.
- Solve: $\tan^2 x \cdot \tan^2 3x \cdot \tan 4x = \tan^2 x - \tan^2 3x + \tan 4x$.

**PRACTICE EXERCISES**

- What are the most general values of θ which satisfy the equations,
 - $\sin \theta = \frac{1}{\sqrt{2}}$
 - $\tan(x - 1) = \sqrt{3}$
 - $\tan \theta = -1$
 - $\operatorname{cosec} \theta = \frac{2}{\sqrt{3}}$
 - $2 \cot^2 \theta = \operatorname{cosec}^2 \theta$
- Solve: $\sin 9\theta = \sin \theta$.
- Solve: $\cot \theta + \tan \theta = 2 \operatorname{cosec} \theta$.
- Solve: $\sin 2\theta = \cos 3\theta$.
- Solve: $\cot \theta = \tan 8\theta$.
- Solve: $\tan^2 \theta - (1 + \sqrt{3}) \tan \theta + \sqrt{3} = 0$.
- Find all the angles between 0° and 90° which satisfy the equation $\sec^2 \theta \cdot \operatorname{cosec}^2 \theta + 2 \operatorname{cosec}^2 \theta = 8$.
- Solve: $4 \cos \theta - 3 \sec \theta = 2 \tan \theta$.
- Solve: $\cot \theta - \tan \theta = 2$.
- Solve: $\sin \theta + \sin 3\theta + \sin 5\theta = 0$.
- Solve: $\cos \theta + \sin \theta = \cos 2\theta + \sin 2\theta$.
- Find all values of θ between 0° and 180° satisfying the equation; $\cos 6\theta + \cos 4\theta + \cos 2\theta + 1 = 0$.
- Solve: $\cos^2 x + \cos^2 2x + \cos^2 3x = 1$.
- Solve: $\sin^2 n\theta - \sin^2(n - 1)\theta = \sin^2 \theta$, where n is constant and $n \neq 0, 1$.

15. Solve: $\sqrt{3} \sin \theta - \cos \theta = \sqrt{2}$.
16. Solve: $\operatorname{cosec} \theta = \cot \theta + \sqrt{3}$.
17. Solve: $5 \sin \theta + 2 \cos \theta = 5$.
18. Solve: $\tan 2\theta \tan \theta = 1$.
19. Solve: $\tan \theta + \tan 2\theta + \sqrt{3} \tan \theta \tan 2\theta = \sqrt{3}$.
20. Solve: $\tan x \cdot \tan \left(x + \frac{\pi}{3}\right) \cdot \tan \left(x + \frac{2\pi}{3}\right) = \sqrt{3}$.
21. If $\tan \theta + \sin \phi = \frac{3}{2}$, $\tan^2 \theta + \cos^2 \phi = \frac{7}{4}$ then find the general value of θ and ϕ .
22. Solve the equation for $0 \leq \theta \leq 2\pi$; $(\sin 2\theta + \sqrt{3} \cos 2\theta)^2 - 5 = \left(\frac{\pi}{6} - 2\theta\right)$.
23. Solve the equation for x , $5^{\frac{1}{2}} + 5^{\frac{1}{2} + \log_5(\sin x)} = 15^{\frac{1}{2} + \log_5 \cos x}$.
24. Find all the values of θ satisfying the equation; $\sin \theta + \sin 5\theta = \sin 3\theta$ such that $0 \leq \theta \leq \pi$.
25. Solve the equality: $2 \sin 11x + \cos 3x + \sqrt{3} \sin 3x = 0$.
26. Find all value of θ , between 0 and π , which satisfy the equation; $\cos \theta \cdot \cos 2\theta \cdot \cos 3\theta = \frac{1}{4}$.
27. Given that A, B are positive acute angle, solve: $\sqrt{3} \sin^2 2A = \sin 2B$ and $\sqrt{3} \sin^2 A + \sin^2 B = \frac{\sqrt{3}-1}{2}$.
28. Find the general solution of the equation, $\tan^2(x+y) + \cot^2(x+y) = 1 - 2x - x^2$.
29. Prove that the equations
(a) $\sin x \cdot \sin 2x \cdot \sin 3x = 1$
(b) $\sin x \cdot \cos 4x \cdot \sin 5x = -\frac{1}{2}$ have no solution
30. Find all the solutions of, $4 \cos^2 x \sin x - 2 \sin^2 x = 3 \sin x$.
31. Solve the inequality $\sin 2x > \sqrt{2} \sin^2 x + (2 - \sqrt{2}) \cos^2 x$.
32. Find the set of values of 'a' for which the equation, $\sin^4 x + \cos^4 x + \sin 2x + a = 0$ possesses solutions. Also find the general solution for these values of 'a'.
33. Find the set of values of x satisfying the equality $\sin \left(x - \frac{\pi}{4}\right) - \cos \left(x + \frac{3\pi}{4}\right) = 1$ and the inequality $\frac{2 \cos 7x}{\cos 3 + \sin 3} > 2^{\cos 2x}$.
34. If 'S' is the solution set in $[0, \pi/2]$ of the trigonometric equation $\cos^6 x + \sin^6 x - \cos^2 2x = \frac{1}{16}$. Then the set 'S' is _____.
35. Solve the inequality $\sin 3x < \sin x$.
36. Find all values of 'a' for which every root of the equation, $a \cos 2x + |a| \cos 4x + \cos 6x = 1$ is also a root of the equation, $\sin x \cos 2x = \sin 2x \cos 3x - \frac{1}{2} \sin 5x$, and conversely, every root of the second equation is also a root of the first equation.
37. Solve the equality: $2 \sin 11x + \cos 3x + \sqrt{3} \sin 3x = 0$
38. Find the general solution of the equation, $2 + \tan x \cdot \cot \frac{x}{2} + \cot x \cdot \tan \frac{x}{2} = 0$
39. Solve for x , the equation $\sqrt{13 - 18 \tan x} = 6 \tan x - 3$, where $-2\pi < x < 2\pi$.
40. Solve the equation for x ,
$$2 \sin \left(3x + \frac{\pi}{4}\right) = \sqrt{1 + 8 \sin 2x \cdot \cos^2 2x}$$



ASSESSMENT

JEE Main

Level I

1. The general solution of the equation, $2 \cos 2x = 3 - 2 \cos^2 x - 4$ is
(A) $x = 2n\pi, n \in I$ (B) $x = n\pi, n \in I$
(C) $x = \frac{n\pi}{4}, n \in I$ (D) $x = \frac{n\pi}{2}, n \in I$
2. The solution set of the equation $4 \sin \theta \cdot \cos \theta - 2 \cos \theta - 2\sqrt{3} \sin \theta + \sqrt{3} = 0$ in the interval $(0, 2\pi)$ is
(A) $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$ (B) $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$
(C) $\left\{\frac{3\pi}{4}, \pi, \frac{5\pi}{3}, \frac{7\pi}{4}\right\}$ (D) $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}\right\}$
3. Total number of solutions of $\sin x \cdot \tan 4x = \cos x$ belonging to $(0, \pi)$ are
(A) 4 (B) 7
(C) 8 (D) 5

4. If $2 \cos^2(\pi+x) + 3 \sin(\pi+x)$ vanishes then the values of x lying in the interval from 0 to 2π are
- (A) $y = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ (B) $x = \frac{\pi}{3}$ or $\frac{5\pi}{3}$
 (C) $x = \frac{\pi}{4}$ or $\frac{5\pi}{4}$ (D) $x = \frac{\pi}{2}$ or $\frac{5\pi}{2}$
5. If $x \in \left[0, \frac{\pi}{2}\right]$, the number of solutions of the equation, $\sin 7x + \sin 4x + \sin x = 0$ is
- (A) 3 (B) 5
 (C) 6 (D) None of these
6. The general solution of $\sin x + \sin 5x = \sin 2x + \sin 4x$ is
- (A) $2n\pi, n \in I$ (B) $n\pi, n \in I$
 (C) $\frac{n\pi}{3}; n \in I$ (D) $2\frac{n\pi}{3}; n \in I$
7. $\frac{\cos 3\theta}{2 \cos 2\theta - 1} = \frac{1}{2}$ if
- (A) $\theta = n\pi + \frac{\pi}{3}, n \in I$
 (B) $\theta = 2n\pi \pm \frac{\pi}{3}, n \in I$
 (C) $\theta = 2n\pi \pm \frac{\pi}{6}, n \in I$
 (D) $\theta = n\pi + \frac{\pi}{6}, n \in I$
8. If $\cos 2\theta + 3 \cos \theta = 0$ then
- (A) $\theta = 2n\pi \pm \alpha$, where $\alpha = \cos^{-1}\left(\frac{\sqrt{17}-3}{4}\right)$
 (B) $\theta = 2n\pi \pm \alpha$, where $\alpha = \cos^{-1}\left(\frac{-\sqrt{17}-3}{4}\right)$
 (C) $\theta = 2n\pi \pm \alpha$, where $\alpha = \cos^{-1}\left(\frac{\pm\sqrt{17}-3}{4}\right)$
 (D) None of these
9. If $\sin \theta + 7 \cos \theta = 5$, then $\tan\left(\frac{\theta}{2}\right)$ is a root of the equation
- (A) $x^2 - 6x + 1 = 0$ (B) $6x^2 - x - 1 = 0$
 (C) $6x^2 + x + 1 = 0$ (D) $x^2 - x + 6 = 0$
10. $\sin 3\theta = 4 \sin \theta \cdot \sin 2\theta \cdot \sin 4\theta$ in $0 \leq \theta \leq \pi$ has
- (A) 2 real solutions (B) 4 real solutions
 (C) 6 real solutions (D) 8 real solutions
11. General solution of the equation, $\cot 3\theta - \cot \theta = 0$ is
- (A) $\theta = (2n-1)\frac{\pi}{2}, n \in I$
 (B) $\theta = (2n-1)\frac{\pi}{4}, n \in I$
 (C) $\theta = (2n-1)\frac{\pi}{3}, n \in I$
 (D) None of these
12. The principal solution set of the equation, $2 \cos x = \sqrt{2+2 \sin 2x}$ is
- (A) $\left\{\frac{\pi}{8}, \frac{13\pi}{8}\right\}$ (B) $\left\{\frac{\pi}{4}, \frac{13\pi}{8}\right\}$
 (C) $\left\{\frac{\pi}{4}, \frac{13\pi}{10}\right\}$ (D) $\left\{\frac{\pi}{8}, \frac{13\pi}{10}\right\}$
13. The number of all possible triplets (a_1, a_2, a_3) such that: $a_1 + a_2 \cos 2x + a_3 \sin^2 x = 0$ for all x is
- (A) 0 (B) 1
 (C) 2 (D) infinite
14. The arithmetic mean of the roots of the equation $4 \cos^3 x - 4 \cos^2 x - \cos(\pi+x) - 1 = 0$ in the interval $[0, 315]$ is equal to
- (A) 49π (B) 50π
 (C) 51π (D) 100π
15. The general solution of $\tan\left(\frac{2}{3}\theta\right) = \sqrt{3}$ is
- (A) $\frac{3n\pi}{2} + \frac{\pi}{2}; n \in I$
 (B) $\frac{n\pi}{2}; \pm \frac{\pi}{2}; n \in I$
 (C) $n\pi \pm \frac{\pi}{2}; n \in I$
 (D) None of these
16. Find the general value of θ , when $\cos\left(\frac{-\theta}{2}\right) = 0$
- (A) $(n+1)\pi; n \in I$
 (B) $n\pi; n \in I$
 (C) $(2n+1)\pi; n \in I$
 (D) $2n\pi; n \in I$

17. If $\tan a\theta - \tan b\theta = 0$, then the values of θ form a series in
 (A) A.P. (B) G.P.
 (C) H.P. (D) None of these
18. The general solution of equation $\sin^2 \theta \sec \theta + \sqrt{3} \tan \theta = 0$ is
 (A) $\theta = n\pi + (-1)^{n+1} \frac{\pi}{3}$
 (B) $\theta = n\pi$
 (C) $\theta = n\pi + (-1)^{n+1} \frac{\pi}{6}$
 (D) $\theta = \frac{n\pi}{2}$
19. If $(1 + \tan \theta)(1 + \tan \phi) = 2$, then $\theta + \phi =$
 (A) 30° (B) 45°
 (C) 60° (D) 75°
20. The general solution of $\sin x + 3 \sin 2x + \sin 3x = \cos x + 3 \cos 2x + \cos 3x$ then in the interval $0 \leq x \leq 2\pi$, $x =$
 (A) $\frac{\pi}{8}, \frac{5\pi}{8}, \frac{2\pi}{3}$
 (B) $\frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$
 (C) $\frac{4\pi}{3}, \frac{9\pi}{3}, \frac{2\pi}{3}, \frac{13\pi}{8}$
 (D) $\frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{3}, \frac{4\pi}{3}$
21. The general value of θ satisfying the equation $\sin^2 \theta - 2 \cos \theta + \frac{1}{4} = 0$
 (A) $2n\pi \pm \frac{\pi}{3}$ (B) $2n\pi \pm \frac{\pi}{4}$
 (C) $2n\pi \pm \frac{\pi}{6}$ (D) None of these
22. If $\tan \theta + \tan 4\theta + \tan 7\theta = \tan \theta \tan 4\theta \tan 7\theta$, then $\theta =$
 (A) $\frac{n\pi}{4}$
 (B) $\frac{n\pi}{7}$
 (C) $\frac{n\pi}{12}; n \neq 6(2k+1)$
 (D) $n\pi$
23. If $3(\sec^2 \theta + \tan^2 \theta) = 5$, then the general value of θ is
 (A) $2n\pi + \frac{\pi}{6}$ (B) $2n\pi \pm \frac{\pi}{6}$
 (C) $n\pi \pm \frac{\pi}{6}$ (D) $n\pi \pm \frac{\pi}{3}$
24. Smallest positive x satisfying the equation $\cos^3 3x + \cos^3 5x = 8 \cos^3 4x \cdot \cos^3 x$ is
 (A) 15° (B) 18°
 (C) 22.5° (D) 30°
25. The solutions of the equation, $(1 + \cos x) \sqrt{\tan \frac{x}{2} - 2} + \sin x = 2 \cos x$ are identical to the solutions of the equation
 (A) $\sin x = 1$ (B) $\cos x = 0$
 (C) $\sin 2x = 0$ (D) $\sec(x/2) = \sqrt{2}$
26. $\sin 3\theta = 4 \sin \theta \sin 2\theta \sin 4\theta$ in $0 \leq \theta \leq \pi$ has
 (A) 2 real solutions (B) 4 real solutions
 (C) 6 real solutions (D) 8 real solutions.
27. The equation, $\sin^2 x - \frac{4}{\sin^3 x - 1} = 1 - \frac{4}{\sin^3 x - 1}$ has
 (A) no root (B) one root
 (C) two roots (D) infinite roots
28. The number of solutions of $\tan(5\pi \cos \theta) = \cot(5\pi \sin \theta)$ for θ in $(0, 2\pi)$ is:
 (A) 28 (B) 14
 (C) 4 (D) 2
29. The value of x satisfying the equation $\sqrt{1 + \sin x} - \sqrt{1 - \sin x} = 2 \sin \frac{x}{2}$ are where $n \in I$
 (A) $2n\pi < x < 2(n+1)\pi$
 (B) $2n\pi - \pi/2 < x < 2n\pi + \pi/2$
 (C) $n\pi - \pi/4 < x < n\pi + \pi/4$
 (D) $(2n-1)\pi/2 < x < (2n+1)\pi$
30. If $x = \frac{n\pi}{2}$, satisfies the equation $\sin \frac{x}{2} - \cos \frac{x}{2} = 1 - \sin x$ and the inequality $\left| \frac{x}{2} - \frac{x}{2} \right| \leq \frac{3\pi}{4}$, then:
 (A) $n = -1, 0, 3, 5$ (B) $n = 1, 2, 4, 5$
 (C) $n = 0, 2, 4$ (D) $n = -1, 1, 3, 5$
31. There is a formula that says that
 $\sin 7x = A \sin^7 x + B \sin^6 x + C \sin^5 x + D \sin^4 x + E \sin^3 x + F \sin^2 x + G \sin x + H$.
 The value of the sum $(A + B + C + D + E + F + G + H)$, is
 (A) 0 (B) 1
 (C) -1 (D) not possible to determine

32. Number of solutions of the equation

$$\cos^2 x + \left(\frac{\sqrt{3}+1}{2}\right) \sin x = \frac{\sqrt{3}}{4} + 1 \text{ in the interval}$$

$[-\pi, \pi]$, is

- (A) 2 (B) 4
(C) 6 (D) 8

33. The expression $\sin x - \cos^2 x - 1$ assumes the least value for the set of real values of x given by

(A) $x = n\pi + (-1)^{n+1} \left(\frac{\pi}{6}\right)$

(B) $x = n\pi + (-1)^n \left(\frac{\pi}{6}\right)$

(C) $x = n\pi + (-1)^n \left(\frac{\pi}{3}\right)$

(D) $x = n\pi - (-1)^n \left(\frac{\pi}{3}\right)$

where $n \in I$

34. The number of roots of the equation, $\sin x + 2 \sin 2x = 3 + \sin 3x$ is

- (A) 0 (B) 1
(C) 2 (D) infinite

35. The smallest positive root of the equation $\sqrt{\sin(1-x)} = \sqrt{\cos x}$ is

(A) $\frac{3\pi}{4} + \frac{1}{2}$ (B) $\frac{7\pi}{4} + \frac{1}{2}$

(C) $\frac{11\pi}{4} + \frac{1}{2}$ (D) $\frac{\pi}{4} + \frac{1}{2}$

Level II

1. All solutions of the equation, $2 \sin \theta + \tan \theta = 0$ are obtained by taking all integral values of m and n in

(A) $2n\pi + \frac{2\pi}{3}, n \in I$

(B) $n\pi$ or $2m\pi \pm \frac{2\pi}{3}$ where $n, m \in I$

(C) $n\pi$ or $m\pi \pm \frac{\pi}{3}$ where $n, m \in I$

(D) $n\pi$ or $2m\pi \pm \frac{\pi}{3}$ where $n, m \in I$

2. The most general solution of $\tan \theta = -1$ and $\cos \theta = \frac{1}{\sqrt{2}}$ is

(A) $n\pi + \frac{7\pi}{4}, n \in I$ (B) $n\pi + (-1)^n \frac{7\pi}{4}, n \in I$

(C) $2n\pi + \frac{7\pi}{4}, n \in I$ (D) None of these

3. A triangle ABC is such that $\sin(2A + B) = \frac{1}{2}$. If A, B, C are in A.P. then the angle A, B, C are, respectively

(A) $\frac{5\pi}{12}, \frac{\pi}{4}, \frac{\pi}{3}$ (B) $\frac{\pi}{4}, \frac{\pi}{3}, \frac{5\pi}{12}$

(C) $\frac{\pi}{3}, \frac{\pi}{4}, \frac{5\pi}{12}$ (D) $\frac{\pi}{3}, \frac{5\pi}{12}, \frac{\pi}{4}$

4. $\frac{\sin 3\theta}{2 \cos 2\theta + 1} = \frac{1}{2}$ if

(A) $\theta = n\pi + \frac{\pi}{6}, n \in I$

(B) $\theta = 2n\pi - \frac{\pi}{6}, n \in I$

(C) $\theta = n\pi + (-1)^n \frac{\pi}{6}, n \in I$

(D) $\theta = n\pi - \frac{\pi}{6}, n \in I$

5. The set of values of x for which $\frac{\tan 3x - \tan 2x}{1 + \tan 3x \tan 2x} = 1$ is

(A) ϕ

(B) $\left(\frac{\pi}{4}\right)$

(C) $\{n\pi + \frac{\pi}{4} \mid n = 1, 2, 3, \dots\}$

(D) $\{2n + \frac{\pi}{4} \mid n = 1, 2, 3, \dots\}$

6. The value 'a' for which the equation $4 \operatorname{cosec}^2(\pi(a+x)) + a^2 - 4a = 0$ has a real solution is:

(A) $a = 1$

(B) $a = 2$

(C) $a = 3$

(D) None of these

7. The solution of $|\cos x| = \cos x - 2 \sin x$ is

(A) $x = n\pi, n \in I$

(B) $x = n\pi + \frac{\pi}{4}, n \in I$

(C) $x = n\pi + (-1)^n \frac{\pi}{4}, n \in I$

(D) $(2n+1)\pi + \frac{\pi}{4}, n \in I$

8. The number of solutions of $\sin \theta + 2 \sin 2\theta + 3 \sin 3\theta + 4 \sin 4\theta = 10$ in $(0, \pi)$ is
 (A) 1 (B) 2
 (C) 4 (D) 0
9. Total number of ordered pairs (x, y) of real numbers satisfying $|\tan \pi y| + (\sin \pi x)^2 = 0$ and $x^2 + y^2 \leq 2$ is equal to
 (A) 3 (B) 6
 (C) 9 (D) 12
10. The solution of the equation $5^{(\sin 2x - \tan x)} - 4^{(5 \cos 2x - \tan x)} = 5^{(1 - \tan x)} (0.04)^{\sin^2 x}$ is
 (A) $(2n+1)\frac{\pi}{2}; m\pi + \frac{\pi}{4}$
 (B) $m\pi + \frac{\pi}{4}$
 (C) $(2n+1)\frac{\pi}{4}$
 (D) all of these
 where $m, n \in I$
11. If $\sum_{n=0}^{\infty} \cos^{2n} \theta = 4$, then the most general values of θ are given by
 (A) $\theta = n\pi \pm \frac{\pi}{3}$ (B) $\theta = 2n\pi \pm \frac{\pi}{4}$
 (C) $\theta = n\pi \pm \frac{\pi}{6}$ (D) $\theta = 2n\pi \pm \frac{\pi}{6}$
 where $n \in I$
12. General solution of the equation $\sec x = 1 + \cos x + \cos^2 x + \cos^3 x + \dots \infty$, is
 (A) $n\pi + \frac{\pi}{3}$ (B) $2n\pi \pm \frac{\pi}{3}$
 (C) $n\pi \pm \frac{\pi}{6}$ (D) $2n\pi \pm \frac{\pi}{6}$
 where n is an integer.
13. Let
$$f(x) = \frac{\cos \operatorname{cosec}^4 x - 2 \cos \operatorname{cosec}^2 x + 1}{\operatorname{cosec} x (\cos \operatorname{cosec} x - \sin x) + \frac{\sin x - \cos x}{\sin x} + \cot x}$$

 The sum of all the solutions of $f(x) = 0$ in $[0, 100\pi]$ is
 (A) 2550π (B) 2500π
 (C) 5000π (D) 5050π
14. If $32 \tan^8 \theta = 2 \cos^2 \alpha - 3 \cos \alpha$ and $3 \cos 2\theta = 1$, then the most general values of α are
 (A) $n\pi - \frac{\pi}{3}$ (B) $2n\pi \pm \frac{2\pi}{3}$
 (C) $n\pi + \frac{\pi}{3}$ (D) $n\pi \pm \frac{\pi}{3}$
 where $n \in I$
15. Let X be the set of all solutions to the equation $\cos x \cdot \sin\left(x + \frac{1}{x}\right) = 0$. Number of real numbers contained by X in the interval $(0 < x < \pi)$, is
 (A) 0 (B) 1
 (C) 2 (D) more than 2
16. If $\tan^2 x = \frac{\sqrt{3}-1}{5+3\sqrt{3}}$, then the general values of x are
 (A) $n\pi \pm \frac{\pi}{12}$ (B) $n\pi \pm \frac{\pi}{8}$
 (C) $n\pi \pm \frac{5\pi}{12}$ (D) $n\pi \pm \frac{3\pi}{8}$
 where $n \in I$
17. The general values of x satisfying simultaneously the equations $2 \sin 2x + \sqrt{3} = 0$ and $\sqrt{3} \tan x + 1 = 0$ is given by
 (A) $2n\pi + \frac{5\pi}{6}, n \in I$
 (B) $2n\pi + \frac{\pi}{6}, n \in I$
 (C) $n\pi + \frac{5\pi}{6}, n \in I$
 (D) $n\pi + \frac{\pi}{6}, n \in I$
18. The true solution set of inequality $\log_2(\sin \theta) > \log_2(\cos \theta)$ is equal to
 (A) $\bigcup_{n \in I} \left(2n\pi + \frac{\pi}{4}, 2n\pi + \frac{5\pi}{4}\right)$
 (B) $\bigcup_{n \in I} \left(2n\pi + \frac{\pi}{4}, 2n\pi + \frac{\pi}{2}\right)$
 (C) $\bigcup_{n \in I} \left(2n\pi, 2n\pi + \frac{\pi}{4}\right)$
 (D) $\bigcup_{n \in I} \left(2n\pi + \frac{\pi}{4}, 2n\pi + \frac{7\pi}{4}\right)$

19. In $\triangle ABC$, if $\cos A + \sin A - \frac{2}{\cos B + \sin B} = 0$, then $\frac{a+b}{c}$ is equal to
 (A) $\sqrt{2}$ (B) 1
 (C) $\frac{1}{\sqrt{2}}$ (D) $2\sqrt{2}$
20. If $0 \leq \theta \leq 2\pi$, then the solution set of the equation $\sin \theta - \sec \theta \cot^3 \theta - \operatorname{cosec} \theta = 0$, is
 (A) ϕ (B) $\{0\}$
 (C) $\left\{\frac{\pi}{2}\right\}$ (D) $\left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

Integer-type Questions

1. Find the principal solution of the trigonometric equation

$$\sqrt{\cot 3x + \sin^2 x - \frac{1}{4}} + \sqrt{\sqrt{3} \cos x + \sin x - 2} = \sin \frac{3x}{2} - \frac{\sqrt{2}}{2}$$

2. Determine the smallest positive value of x which satisfy the equation, $\sqrt{1 + \sin 2x} - \sqrt{2} \cos 3x = 0$.

3. Find the number of solutions of the equation $\sin 2x + \cos 2x + 4 \sin x = 1 + 4 \cos x$ lying in interval $[-6, 6]$.
4. Find the number of solutions of the equation $\prod_{r=1}^{12} \sin rx = 0$ lying in the interval $(0, \pi]$.
5. Sum of all the solutions in $[0, 4\pi]$ of the equation $\tan x + \cot x + 1 = \cos\left(x + \frac{\pi}{4}\right)$ is $k\pi$ then the value of k is
6. If the quadratic equation $x^2 + (2 - \tan \theta)x - (1 + \tan \theta) = 0$ has two integral roots, then sum of all possible values of θ in interval $(0, 2\pi)$ is $k\pi$. Find the value of k .
7. If the values of y satisfying the equation $x^2 - 2x \sin(xy) + 1 = 0$ is expressed in the form of $k\pi$ ($k \in \mathbb{R}$) then find the sum of all possible values of k in $(0, 48)$.
8. Find the number of ordered pairs (x, y) satisfying the equations $\sin x \cos y = 1$ and $x^2 + y^2 \leq 9\pi^2$.
9. Find the number of solutions of the equation $\sin^2 2x - \cos^2 8x = \frac{1}{2} \cos 10x$ lying in the interval $\left(0, \frac{\pi}{2}\right)$.

JEE Advanced

Single Correct Option-type Questions

1. If $20 \sin^2 \theta + 21 \cos \theta - 24 = 0$ and $\frac{7\pi}{4} < \theta < 2\pi$ then the values of $\cot \frac{\theta}{2}$ is
 (A) 3 (B) $\frac{\sqrt{15}}{3}$
 (C) $-\frac{\sqrt{15}}{3}$ (D) -3
2. The general solution of the equation $\tan x + \tan\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right) = 3$ is
 (A) $\frac{n\pi}{4} + \frac{\pi}{12}, n \in I$ (B) $\frac{n\pi}{3} + \frac{\pi}{6}, n \in I$
 (C) $\frac{n\pi}{3} + \frac{\pi}{12}, n \in I$ (D) None of these

3. The general solution of the equation $\tan^2 \alpha + 2\sqrt{3} \tan \alpha = 1$ is given by
 (A) $\alpha = \frac{n\pi}{2}, n \in I$
 (B) $\alpha = (2n + 1)\frac{\pi}{2}, n \in I$
 (C) $\alpha = (6n + 1)\frac{\pi}{12}, n \in I$
 (D) $\alpha = \frac{n\pi}{12}, n \in I$
4. The number of integral values of a for which the equation $\cos 2x + a \sin x = 2a - 7$ possesses a solution is
 (A) 2 (B) 3
 (C) 4 (D) 5
5. The number of solution of the equation $|\sin x| = |\cos 3x|$ in $[-2\pi, 2\pi]$ is
 (A) 32 (B) 28
 (C) 24 (D) 30

6. If $2 \tan^2 x - 5 \sec x - 1 = 0$ has 7 different roots in $\left[0, \frac{n\pi}{2}\right]$, $n \in \mathbb{N}$, then greatest value of n is
 (A) 8 (B) 10
 (C) 13 (D) 15
7. The values of x between 0 and 2π which satisfy the equation $\sin x \cdot \sqrt{8 \cos^2 x} = 1$ are in A.P. The common difference of the A.P. is
 (A) $\frac{\pi}{8}$ (B) $\frac{\pi}{4}$
 (C) $\frac{3\pi}{8}$ (D) $\frac{3\pi}{4}$
8. Number of solution of the equation $\frac{3 \sin x - \sin 3x}{1 + \cos \theta} + \frac{3 \cos x + \cos 3x}{1 - \sin \theta} = 4\sqrt{2} \cos\left(\theta + \frac{\pi}{4}\right)$ in the interval $(-10\pi, 8\pi]$ is equal to
 (A) 8 (B) 9
 (C) 16 (D) 18
9. If the system of equations $x - y = 2$ and $cx + y = 3$ have a solution (x, y) where $x > 0$ and $y > 0$, then the true set of values of c is
 (A) $c > -1$ (B) $c < \frac{3}{2}$
 (C) $0 < c < \frac{3}{2}$ (D) $-1 < c < \frac{3}{2}$
10. The number of ordered pair(s) (x, y) of real numbers satisfying the equation $1 + x^2 + 2x \sin(\cos^{-1} y) = 0$, is
 (A) 0 (B) 1
 (C) 2 (D) 3
11. Number of solution of the equation $2\sqrt{\cos^6 \theta - \sin^6 \theta} = 1 + \cos 2\theta$ in $[-\pi, \pi]$ is
 (A) 1 (B) 2
 (C) 3 (D) 4
12. The true solution set of the inequality $\sin x + \sin 2x \geq 0$ in $(0, 2\pi)$ is
 (A) $\left(0, \frac{2\pi}{3}\right] \cup \left[\pi, \frac{4\pi}{3}\right)$
 (B) $\left(0, \frac{\pi}{3}\right] \cup \left[\frac{4\pi}{3}, 2\pi\right)$
 (C) $\left[\frac{2\pi}{3}, \frac{4\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 2\pi\right)$
 (D) $[0, \pi] \cup \left[\frac{5\pi}{3}, 2\pi\right)$
13. The values of x for which $\min(\sin x, \cos x) > \min(-\sin x, -\cos x)$ where $x \in (0, 2\pi)$ equals
 (A) $(0, \pi)$ (B) $\left(\frac{3\pi}{4}, 2\pi\right)$
 (C) $\left(0, \frac{5\pi}{6}\right)$ (D) $\left(\frac{7\pi}{4}, 2\pi\right) \cup \left(0, \frac{3\pi}{4}\right)$
14. General solution of the equation $|\cos x| = \sin x$, is
 (A) $n\pi + (-1)^n \frac{\pi}{4}$ (B) $2n\pi \pm \frac{\pi}{4}$
 (C) $n\pi + \frac{\pi}{4}$ (D) $2n\pi + \frac{\pi}{4}$
 where $n \in \mathbb{I}$
15. Number of solutions of the equation $2 \tan^2 x - 5 \sec x = 1$ in $(-\pi, 2\pi)$ is equal to
 (A) 2 (B) 3
 (C) 4 (D) 5
16. General solution of the trigonometric equation, $\left(2 \sin^4 \frac{x}{2} - 1\right) \frac{1}{\cos^4 \frac{x}{2}} = 2$ is
 (A) $n\pi + \frac{2\pi}{3}$ (B) $n\pi \pm \frac{\pi}{3}$
 (C) $2n\pi + \frac{2\pi}{3}$ (D) $2n\pi \pm \frac{2\pi}{3}$
17. Let ABC be an acute triangle with all sides unequal. D, E and F are the feet of the perpendiculars from the vertex A, B and C respectively. The circumcircle of the triangle AEF passes through the
 (A) centroid of the ΔABC .
 (B) orthocentre of the ΔABC .
 (C) incentre of the ΔABC .
 (D) circumcentre of the ΔABC .
18. Number of ordered pairs (x, y) of real numbers satisfying the system of equations $\sin x = \sin 2y$ and $\cos x = \sin y$ where $0 \leq x \leq \pi$ and $0 \leq y \leq \pi$, is
 (A) 1 (B) 2
 (C) 3 (D) 4

19. The true set of values of $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ satisfying $|4\sin x - 1| < \sqrt{5}$ is equal to
- (A) $\left(\frac{\pi}{10}, \frac{3\pi}{10}\right)$ (B) $\left(\frac{-\pi}{10}, \frac{3\pi}{10}\right)$
 (C) $\left(\frac{-\pi}{5}, \frac{3\pi}{5}\right)$ (D) $\left(\frac{-2\pi}{3}, \frac{\pi}{3}\right)$
20. If $2^{\sqrt{\sin^2 x - 2\sin x + 5} - 2\sin^2 y} \leq 1$ then number of ordered pairs (x, y) in $[0, 2\pi]$
- (A) 0 (B) 1
 (C) 2 (D) 3
21. Complete set of values of 'a' for which a $\sin^2 x + |\cos x| - 2a = 0$ has atleast one solution, is
- (A) $\left[\frac{-1}{2}, 0\right)$ (B) $\left[0, \frac{1}{2}\right]$
 (C) $\left(0, \frac{1}{2}\right]$ (D) $\left(0, \frac{1}{2}\right)$
22. If the equation $p \sin 2x + \cos x = p$ has a solution, then the range of p is
- (A) $(-\infty, \infty)$ (B) $[-1, 1]$
 (C) $(-\infty, 0]$ (D) $[0, \infty)$
23. Given $0 \leq \theta \leq 2\pi$, the number of possible solutions satisfying the equation $8 \cos^6 \theta - 7 \cos^3 2\theta - 1 = 0$, is
- (A) 5 (B) 6
 (C) 7 (D) 8
24. If the expression $E = \sin \frac{\theta}{2} - \sqrt{3} \cos \frac{\theta}{2}$ ($\theta \in R$) attains its maximum value then the most general values of θ are given by
- (A) $4k\pi + \frac{5\pi}{3}$ (B) $2k\pi + \frac{5\pi}{3}$
 (C) $4k\pi + \frac{5\pi}{6}$ (D) $2k\pi + \frac{5\pi}{6}$
 (where $k \in I$)
25. Sum of all the solutions of the equation $\cos 2x + \sin^2 2x = 1$, which lie in the interval $[0, 2\pi]$ is equal to
- (A) 4π (B) 5π
 (C) 7π (D) 8π
26. Given $\frac{\cos^2 \theta - \sin^2 \theta}{2 \cos^2 \theta \tan \theta} = 2 - \sqrt{3}$ then the value of $\theta \in (0, \pi/2)$ is
- (A) $\frac{\pi}{24}$ (B) $\frac{5\pi}{12}$
 (C) $\frac{5\pi}{24}$ (D) $\frac{\pi}{12}$
27. Let $\theta \in [0, 4\pi]$ satisfying the equation $(\sin \theta + 2)(\sin \theta + 3)(\sin \theta + 4) = 6$. If the sum of all value of θ is of the form $k\pi$ then the value of 'k', is
- (A) 6 (B) 5
 (C) 4 (D) 2
28. Number of solution of the equation, $\sin^4 x - \cos^2 x \sin x + 2\sin^2 x + \sin x = 0$ in $0 \leq x \leq 3\pi$, is
- (A) 3 (B) 4
 (C) 5 (D) 6

Multiple Correct Option-type Questions

1. $\sin x, \sin 2x, \sin 3x$ are in A.P. if
- (A) $x = \frac{n\pi}{2}, n \in I$ (B) $x = n\pi, n \in I$
 (C) $x = 2n\pi, n \in I$ (D) $x = (2n + 1)\pi, n \in I$
2. $\sin x - \cos^2 x - 1$ assumes the least value for the set of values of x given by
- (A) $x = n\pi + (-1)^{n+1} \left(\frac{\pi}{6}\right), n \in I$
 (B) $x = n\pi + (-1)^n \left(\frac{\pi}{6}\right), n \in I$
 (C) $x = n\pi + (-1)^n \left(\frac{\pi}{3}\right), n \in I$
 (D) $x = n\pi - (-1)^n \left(\frac{\pi}{6}\right), n \in I$
3. $\sin x + \sin 2x + \sin 3x = 0$ if
- (A) $\sin x = \frac{1}{2}$ (B) $\sin 2x = 0$
 (C) $\sin 3x = \frac{\sqrt{3}}{2}$ (D) $\cos x = -\frac{1}{2}$
4. $\cos 4x \cos 8x - \cos 5x \cos 9x = 0$ if
- (A) $\cos 12x = \cos 14x$
 (B) $\sin 13x = 0$
 (C) $\sin x = 0$
 (D) $\cos x = 0$

5. The general solution of the equation $\cos x \cdot \cos 6x = -1$, is
 (A) $x = (2n + 1)\pi, n \in I$
 (B) $x = 2n\pi, n \in I$
 (C) $x = (2n - 1)\pi, n \in I$
 (D) None of these
6. If $\sin(x - y) = \cos(x + y) = \frac{1}{2}$ then the values of x and y lying between 0 and π are given by
 (A) $x = \frac{\pi}{4}, y = \frac{3\pi}{4}$ (B) $x = \frac{\pi}{4}, y = \frac{\pi}{12}$
 (C) $x = \frac{5\pi}{4}, y = \frac{5\pi}{12}$ (D) $x = \frac{11\pi}{12}, y = \frac{3\pi}{4}$
7. The equation $2 \sin \frac{x}{2} \cdot \cos^2 x + \sin^2 x = 2 \sin \frac{x}{2} \cdot \sin^2 x + \cos^2 x$ has a root for which
 (A) $\sin 2x = 1$ (B) $\sin 2x = -1$
 (C) $\cos x = \frac{1}{2}$ (D) $\cos 2x = -\frac{1}{2}$
8. $\cos 15x = \sin 5x$ if
 (A) $x = -\frac{\pi}{20} + \frac{n\pi}{5}, n \in I$
 (B) $x = \frac{\pi}{40} + \frac{n\pi}{10}, n \in I$
 (C) $x = \frac{3\pi}{20} + \frac{n\pi}{5}, n \in I$
 (D) $x = -\frac{3\pi}{40} + \frac{n\pi}{10}, n \in I$
9. $5 \sin^2 x + \sqrt{3} \sin x \cos x + 6 \cos^2 x = 5$ if
 (A) $\tan x = \frac{-1}{\sqrt{3}}$
 (B) $\sin x = 0$
 (C) $x = n\pi + \frac{\pi}{2}, n \in I$
 (D) $x = n\pi + \frac{\pi}{6}, n \in I$
10. $\sin^2 x + 2 \sin x \cos x - 3 \cos^2 x = 0$ if
 (A) $\tan x = 3$
 (B) $\tan x = -1$
 (C) $x = n\pi + \frac{\pi}{4}, n \in I$
 (D) $x = n\pi + \tan^{-1}(-3), n \in I$
11. $\sin^2 x - \cos 2x = 2 - \sin 2x$ if
 (A) $x = \frac{n\pi}{2}, n \in I$
 (B) $\tan x = \frac{3}{2}$
 (C) $x = (2n + 1) \frac{\pi}{2}, n \in I$
 (D) $x = n\pi + (-1)^n \sin^{-1} \left(\frac{2}{3} \right), n \in I$
12. Which of the following equation(s) have NO solution?
 (A) $\sin x + \sin 9x = 2$
 (B) $\cos x - \sin 3x = -2$
 (C) $\sin \left(\frac{5x}{2} \right) - \sin \left(\frac{x}{2} \right) = 2$
 (D) $\cos x \cos 6x = -1$
13. Let $2 \sin x + 3 \cos y = 3$ and $3 \sin y + 2 \cos x = 4$ then
 (A) $x + y = (4n + 1)\pi/2, n \in I$
 (B) $x + y = (2n + 1)\pi/2, n \in I$
 (C) x and y can be the two non right angles of a 3-4-5 triangle with $x > y$.
 (D) x and y can be the two non right angles of a 3-4-5 triangle with $y > x$.
14. The equation $\operatorname{cosec} x + \sec x = 2\sqrt{2}$ has
 (A) no solution in $\left(0, \frac{\pi}{4} \right)$
 (B) a solution in $\left[\frac{\pi}{4}, \frac{\pi}{2} \right)$
 (C) no solution in $\left(\frac{\pi}{2}, \frac{3\pi}{4} \right)$
 (D) a solution in $\left[\frac{3\pi}{4}, \pi \right)$
15. In which of the following interval(s) the value of the expression $y = 2(\sqrt{2} - 1) \sin x - 2 \cos 2x + 2 - \sqrt{2}$ is negative?
 (A) $\left(\frac{\pi}{6}, \frac{\pi}{2} \right)$ (B) $\left(0, \frac{\pi}{6} \right)$
 (C) $\left(\frac{5\pi}{6}, \frac{5\pi}{4} \right)$ (D) $\left(\frac{7\pi}{4}, 2\pi \right)$
16. The equation $\sin \alpha = \tan(\alpha - \beta) + \cos \alpha \cdot \tan \beta$ holds true if
 (A) $\alpha = n\pi + \beta$
 (B) $\alpha = 2\nu\pi + 2\beta$
 (C) $\alpha = 2n\pi$ and $\beta \in R$
 (D) $\beta = 2\nu\pi$ and $\alpha \in R$
 n is an integer

17. If $\cos 3\theta = \cos 3\alpha$ then the value of $\sin \theta$ can be given by

- (A) $\pm \sin \alpha$ (B) $\sin\left(\frac{\pi}{3} \pm \alpha\right)$
 (C) $\sin\left(\frac{2\pi}{3} + \alpha\right)$ (D) $\sin\left(\frac{2\pi}{3} - \alpha\right)$

18. Which of the following sets can be the subset of the general solution of the equation $1 + \cos 3x = 2 \cos 2x$?

- (A) $n\pi + \frac{\pi}{3}$ (B) $n\pi + \frac{\pi}{6}$
 (C) $n\pi - \frac{\pi}{6}$ (D) $2n\pi$

19. Which of the following equations have no solution?

- (A) $2^{|x|} = \sin x^2$ (B) $3^{|\sin \sqrt{x}|} = |\cos x|$
 (C) $x^2 = -\cos x$ (D) $3x^2 = 1 - 2\cos x$

20. Which of the following equations have their general solution equal to $2n\pi$ where $n \in I$?

- (A) $\sin 3x + 3 \sin 4x + \sin 5x = 0$
 (B) $2 \tan^2 x + 3 = \frac{3}{\cos x}$
 (C) $\cos x \cdot \cos 2x \cdot \cos 4x = 1$
 (D) $\sin \frac{3x}{2} \cdot \cos \frac{x}{2} = \frac{1}{2} \sin 2x$

Comprehension-based Questions

Paragraph for question nos. 1 to 3

Equation of the form $P(\sin x \pm \cos x, \sin x \cos x) = 0$ where $P(y, z)$ is a polynomial, can be solved by the change:

$\cos x \pm \sin x = t$; $1 \pm 2 \sin x \cos x = t^2$. Reduce the given

equation into $P\left(t, \frac{t^2 - 1}{2}\right) = 0$.

1. $\sin x + \cos = 1 + \sin x \cos x$ is

- (A) $\frac{\pi}{2} + 2\pi n$ and $2n\pi$
 (B) $\frac{\pi}{4} + 2\pi n$ and $(2n + 1)\pi$
 (C) $2n\pi$
 (D) None of these

2. If $(\cos x - \sin x) \left(2 \tan x + \frac{1}{\cos x} \right) + 2 = 0$, then x is

- (A) $n\pi \pm \frac{\pi}{3}$ (B) $2n\pi \pm \frac{\pi}{3}$
 (C) $n\pi \pm \frac{\pi}{6}$ (D) $2n\pi \pm \frac{\pi}{6}$

3. $\sin^4 x + \cos^4 x = \sin x \cos x$ then x is

- (A) $(6n + 1) \frac{\pi}{6}$ (B) $n\pi$
 (C) $(4n + 1) \frac{\pi}{4}$ (D) None of these

Paragraph for question nos. 4 to 6

Consider an equation $\sin x + \sin y \leq 2$ (1)

We know that $\sin x \leq 1$ and $\sin y \leq 1$ for all x, y .

So $\sin x + \sin y \leq 2$ for all x and y .

Therefore, $\sin x + \sin y = 2$ if and only if $\sin x = 1$ and $\sin y = 1$

$$\Rightarrow x = 2n\pi + \frac{\pi}{2} \text{ and } y = 2m\pi + \frac{\pi}{2}$$

Which is the required solution of given equation. To solve the equation (1), we have used the boundedness of $\sin x$ rather than using conventional methods of solving equations. In general we employ one or more of the following extreme value conditions.

- (a) $-1 \leq \sin x \leq 1 \Rightarrow |\sin x| \leq 1$ and $\sin^2 x \leq 1$
 (b) $-1 \leq \cos x \leq 1 \Rightarrow |\cos x| \leq 1$ and $\cos^2 x \leq 1$
 (c) $-\sqrt{a^2 - b^2} \leq a \sin x + b \cos x \leq \sqrt{a^2 + b^2}$
 $\Rightarrow |a \sin x + b \cos x| \leq \sqrt{a^2 + b^2}$

4. The minimum value of $27^{\cos 2x} 81^{\sin 2x}$ is

- (A) 1 (B) $\frac{1}{9}$
 (C) $\frac{1}{81}$ (D) $\frac{1}{243}$

5. Number of roots of the equation $\cos^7 x + \sin^4 x = 1$ in the interval $[0, 2\pi]$ is

- (A) 0 (B) 1
 (C) 2 (D) 4

6. The smallest positive number of p for which the equation $\cos(p \sin x) = \sin(p \cos x)$ has a solution in $[0, 2\pi]$ is

- (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$
 (C) $\frac{\pi}{4\sqrt{2}}$ (D) $\frac{\pi}{2\sqrt{2}}$

Paragraph for question nos. 7 to 9

Let α, β, γ be positive integers in arithmetic progression such that $\alpha^\circ < \beta^\circ < \gamma^\circ < 180^\circ$.

Also $\sin \alpha^\circ + \sin \beta^\circ = \sin \gamma^\circ$ and $\cos \alpha^\circ - \cos \beta^\circ = \cos \gamma^\circ$.

7. The value of $(\tan \alpha^\circ + \tan \beta^\circ + \tan \gamma^\circ)$ is equal to
 (A) 3 (B) $2 - \sqrt{3}$
 (C) 5 (D) $2\sqrt{2} + 1$
8. Common difference of an arithmetic progression lies in the interval
 (A) (2, 10) (B) (10, 18)
 (C) (20, 25) (D) (25, 35)
9. Number of solutions of the equation $\cos x + \cos 2x + \cos 3x = \tan \alpha^\circ + \tan \gamma^\circ - \tan \beta^\circ$ in the interval $[0, 4\pi]$ is equal to
 (A) 2 (B) 3
 (C) 4 (D) 5

Paragraph for question nos. 10 and 11

Let $f(x) = \cos x + \sin x - 1$ and $g(x) = \sin 2x - 2$.

10. Number of solutions of the equation $f(x) = g(x)$ in $x \in \left[-\pi, \frac{7\pi}{2}\right]$ is equal to
 (A) 4 (B) 5
 (C) 6 (D) 8
11. If the equation $f(x) = k$ has atleast one real solution then the number of possible integral values of k is equal to
 (A) 1 (B) 2
 (C) 3 (D) 4

Matrix Match-type Questions

1. Match the following

Column - I	Column - II
(A) $(2 \sin x - \cos x)(1 + \cos x) = \sin^2 x$	(P) $\sin x = \frac{1}{2}$
(B) $1 + \sin 2x = \cos x + \sin x$	(Q) $\tan x = -1$
(C) $4x^4 + x^6 + \sin^2 5x = 0$	(R) $x = 0$
(D) $\tan x = \frac{1}{\sqrt{3}}$	(S) $x = \frac{19\pi}{5}$

2. Match the following for number of solutions in $[0, 2\pi]$

Column - I	Column - II
(A) $\sin^2 \theta - \tan^2 \theta = 1$	(P) 2
(B) $\sin \theta + \cos \theta = 1$	(Q) 0
(C) $\tan \theta + \sec \theta = 2 \cos \theta$	(R) 3
(D) $3 \sin^2 \theta - 4 \sin \theta + 1 = 0$	(S) 1

3. Match the following

Column - I	Column - II
(A) If $x \in \left[0, \frac{\pi}{2}\right]$, the number of solutions of the equation, $\sin 7x + \sin 4x + \sin x = 0$ is	(P) 4
(B) Let $f(x) = \cos^2 x + \cos^2 2x + \cos^2 3x$. Number of values of $x \in [0, \pi]$ for which $f(x)$ equals the smallest positive integer is	(Q) 5
(C) Number of principal solution of the trigonometric equation $\tan x + \tan 2x + \tan x \cdot \tan 2x = 1$, is	(R) 6
(D) The number of points $P(x, y)$ lying inside or on the circle $x^2 + y^2 = 9$ and satisfying the equation $\tan^4 x + \cot^4 x + 2 = 4 \sin^2 y$, is	(S) 7
(E) Let $0 < \theta_1 < \theta_2 < \theta_3 < \dots$ denote the positive solution of the equation $3 + 3 \cos \theta = 2 \sin^2 \theta$. If $\theta_3 + \theta_7 = a\pi$, where a is an integer then ' a ' equals	(T) 8

4. Match the following

Column - I	Column - II
(A) If solution of the equation $x^4 + \log x = 32$ (where base of logarithm is 2) are x_1 and x_2 ($x_1 > x_2$) then the value of $\frac{x_1}{x_2}$ is divisible by	(P) 2 (Q) 3
(B) Let $\cos(\alpha + \beta) = \frac{4}{5}$; $\sin(\alpha - \beta) = \frac{5}{13}$ and α, β lie between 0 and $\frac{\pi}{4}$. If the value of $\tan 2\alpha$ is $\frac{56}{k}$, then k is divisible by	(R) 8
(C) The difference between maximum and minimum values of $\sin^2 x - 20 \cos x + 1$ is divisible by	(S) 11

5. Match the following

Column - I	Column - II
(A) $\sin x \cdot \cos^3 x > \cos x \cdot \sin^3 x$, $0 \leq x \leq 2\pi$, is	(P) $\left[-\pi, -\frac{3\pi}{4}\right] \cup \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4}, \pi\right]$
(B) $4 \sin^2 x - 8 \sin x + 3 \leq 0$, $0 \leq x \leq 2\pi$, is	(Q) $\left[\frac{3\pi}{2}, 2\pi\right] \cup \{0\}$
(C) $ \tan x \leq 1$ and $x \in [-\pi, \pi]$ is	(R) $\left(0, \frac{\pi}{4}\right)$
(D) $\cos x - \sin x \geq 1$ and $0 \leq x \leq 2\pi$	(S) $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$

Integer-type Questions

- For $0 < x, y < \pi$, find the number of ordered pairs (x, y) satisfying the system of equations $\cot^2(x-y) - (1+\sqrt{3})\cot(x-y) + \sqrt{3} = 0$ and $\cos y = \frac{\sqrt{3}}{2}$.
- Find number of solutions of the equation $(\log_2 \cos \theta)^2 + \log_{\frac{4}{\cos \theta}}(16 \cos \theta) = 2$, in interval $[0, 2\pi)$.
- For $\alpha \in \left(\frac{3\pi}{4}, \pi\right)$ and $\beta \in R - (2n+1)\frac{\pi}{2}$, $n \in I$, find number of ordered pairs (α, β) satisfying the equation $\sqrt{2 \cot \alpha + \frac{1}{\sin^2 \alpha}} = \frac{1 + \cot \alpha}{\sqrt{\tan \beta}}$.
- Find the number of principal solutions of the trigonometric equation $5 \left(\sin x + \frac{\cos 3x + \sin 3x}{1 + 2 \sin 2x} \right) = \cos 2x + 3$.
- Find the number of points of intersection of the curves $y = \cos x$, $y = \sin 3x$, if $\frac{-\pi}{2} \leq x \leq \frac{\pi}{2}$.
- Let N be the number of triplets (x, y, z) where $x, y, z \in [0, 2\pi]$ satisfying the inequality $(4 + \sin 4x)(2 + \cot^2 y)(1 + \sin^4 z) \leq 12 \sin^2 z$. Find the value of $\frac{N}{2}$.
- Let $A = \{\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_m\}$, $B = \{\mu_1, \mu_2, \mu_3, \dots, \mu_n\}$ are two sets of values of λ and μ where $(\lambda_i < \lambda_{i+1}, \mu_j < \mu_{j+1} \forall i$ and j and $\lambda, \mu \in (0, 100\pi]$) such that for the ordered pair (λ_i, μ_j) the equation $x^2 (\sin \lambda x + \cos \mu x) - 2(\sin(\lambda + \mu)x + \sin(\lambda - \mu)x) + \sin \lambda x + \cos \mu x = 0$ has a positive solution. If $\sum_{i=1}^m \lambda_i + \sum_{j=1}^n \mu_j = \frac{5}{2} k\pi$ then find the value of k .
- Consider an equation with x as a variable $7 \sin 3x - 2 \sin 9x = \sec^2 \theta + 4 \operatorname{cosec} 2\theta$, then find the value of $\frac{15}{\pi}$ ((minimum positive root) - (maximum negative root)).
- Find the sum of the first 100 rational terms of the increasing sequence of the positive roots of the equation $\sin(\pi x^2) - \sin(\pi(x^2 + 2x)) = 0$.
- If the sum of all the angles which lie in $[0, 2\pi]$ and satisfying the equation $(8 \cos 4\theta - 3)(\cot \theta + \tan \theta - 2)(\cot \theta + \tan \theta + 2) = 12$ is $k\pi$, find the value of k .
- Find the number of solutions satisfying the equation $\cos \frac{x}{2} - \sin \frac{x}{2} = \sin x - 1$, where $|x - \pi| \leq \frac{3\pi}{2}$.
- Let U : denotes the value of $4 \cos \frac{\pi}{5} - 4 \cos \frac{2\pi}{5} + 4 \sin \frac{\pi}{10} \cos \frac{\pi}{5}$ and V : denotes the number of values of $\theta \in [-2\pi, 2\pi]$ satisfying the equation $\operatorname{cosec} \theta = 1 + \cot \theta$. Find $(U + V)$.
- Find the number of solutions of the equation $2 \cos 3x(3 - 4 \sin^2 x) = 1$ in $[0, 2\pi]$.
- If the sum of all values of x satisfying the system of equations $\tan x + \tan y + \tan x \cdot \tan y = 5$
 $\sin(x + y) = 4 \cos x \cdot \cos y$ is $\frac{k\pi}{2}$, where $x \in \left(0, \frac{\pi}{2}\right)$ then find the value of k .
- Find the number of values of θ , $0 \leq \theta \leq 2\pi$ such that the graphs of $f(x) = \left(2 \sin^2 \frac{\theta}{2}\right)x^2 + \left(\cot \frac{\theta}{2}\right)x - 1$ and $g(x) = \left(2 \cos^2 \frac{\theta}{2}\right)x^2 - \left(\tan \frac{\theta}{2}\right)x + \cot^2 \theta$ has exactly one point in common.

16. Let $x_1, x_2, x_3, \dots, x_n$ be all solutions of the equation $(\sin 3x)(\sin 3x - \cos x) = \sin x(\sin x - \cos 3x)$ where $0 \leq x_i \leq 2\pi \forall i$. If $\sum_{i=1}^n x_i = \frac{a\pi}{b}$ (where a and b are coprime) then find the value of $(a + b)$.
17. If the sum of all the principal solutions of the equation $2x - \tan \frac{\pi}{6} \cos 2x = 1$ is $\frac{k\pi}{12}$, then find the value of k .
18. Find the number of distinct solutions for the equation $\cos x \cos 2x \cos 3x \cos 4x \cos 5x = 0$ in the interval $[0, \pi]$.
19. Find the number of solutions of the equation $\sin 2\theta + \cos 2\theta + 4 \sin \theta = 1 + 4 \cos \theta$ lying in the interval $[-2\pi, 2\pi]$.
20. Sum of the principal solutions of the equation, $\sec x + \tan x = 2 \cos x$ is _____.
21. Find the smallest positive number p for which the equation: $\cos(p \sin x) = \sin(p \cos x)$ has a solution $x \in [0, 2\pi]$.
22. Determine the smallest positive value of x which satisfy the equation, $\sqrt{1 + \sin 2x} - \sqrt{2} \cos 3x = 0$.



JEE ARCHIVES

JEE Main

1. Find the number of roots of the equation $\tan x + \sec x = 2 \cos x$ in the interval $[0, 2\pi]$ [AIEEE-2002]
 (A) 1 (B) 2
 (C) 3 (D) 4
2. General solution of $\tan 5\theta = \cot 2\theta$ is [AIEEE-2002]
 (A) $\theta = \frac{n\pi}{7} + \frac{\pi}{14}$
 (B) $\theta = \frac{n\pi}{7} + \frac{\pi}{5}$
 (C) $\theta = \frac{n\pi}{7} + \frac{\pi}{2}$
 (D) $\theta = \frac{n\pi}{7} + \frac{\pi}{5}, n \in Z$
3. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation $2 \sin^2 x + 5 \sin x - 3 = 0$ is [AIEEE-2006]
 (A) 6 (B) 1
 (C) 2 (D) 4
4. If $0 < x < \pi$, and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is [AIEEE-2006]
 (A) $\frac{(4 - \sqrt{7})}{3}$ (B) $-\frac{(4 + \sqrt{7})}{3}$
 (C) $\frac{(1 + \sqrt{7})}{4}$ (D) $\frac{(1 - \sqrt{7})}{4}$
5. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has [AIEEE-2012]
 (A) exactly one real root
 (B) exactly four real root
 (C) infinite number of real root
 (D) no real root
6. In ΔPQR if $3 \sin P + 4 \cos Q = 6$ and $4 \sin Q + 3 \cos P = 1$, then the angle R is equal to [AIEEE-2012]
 (A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$
 (C) $\frac{5\pi}{6}$ (D) $\frac{\pi}{6}$
7. If $0 \leq x < 2\pi$, then the number of real values of x , which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$, is [JEE-2016]
 (A) 5 (B) 7
 (C) 9 (D) 3
8. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30° . After walking for 10 minutes from A in the same direction, at a point B , he observes that the angle of elevation of the top of the pillar is 60° . Then the time taken (in minutes) by him, from B to reach the pillar, is [JEE-2016]
 (A) 10 (B) 20
 (C) 5 (D) 6

9. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of $\cos 4x$ is [JEE-2017]
 (A) $-\frac{7}{9}$ (B) $-\frac{3}{5}$
 (C) $\frac{1}{3}$ (D) $\frac{2}{9}$
10. If sum of all the solutions of the equation $8\cos x \cdot \left(\cos\left(\frac{\pi}{6} + x\right) \cdot \cos\left(\frac{\pi}{6} - x\right) - \frac{1}{2}\right) = 1$ in $[0, \pi]$ is $k\pi$, then k is equal to [JEE-2018]
 (A) $\frac{20}{9}$ (B) $\frac{2}{3}$
 (C) $\frac{13}{9}$ (D) $\frac{8}{9}$
11. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ for $k = 1, 2, 3, \dots$. Then for all $x \in R$, the value of $f_4(x) - f_6(x)$ is equal to [JEE-2019]
 (A) $1/12$ (B) $1/4$
 (C) $-1/12$ (D) $5/12$
12. If $\sin^4 \alpha + 4\cos^4 \beta + 2 = 4\sqrt{2} \sin \alpha \cos \beta$; $\alpha, \beta \in [0, \pi]$, then $\cos(\alpha + \beta) - \cos(\alpha - \beta)$ is equal to [JEE-2019]
 (A) -1 (B) 0
 (C) $-\sqrt{2}$ (D) $\sqrt{2}$
13. $\left(\frac{1 + \sin \frac{2\pi}{9} + i \cos \frac{2\pi}{9}}{1 + \sin \frac{2\pi}{9} - i \cos \frac{2\pi}{9}}\right)^3$ is [JEE-2020]
 (A) $-\frac{1}{2}(1 - i\sqrt{3})$ (B) $-\frac{1}{2}(1 + i\sqrt{3})$
 (C) $\frac{1}{2}(1 - i\sqrt{3})$ (D) $\frac{1}{2}(\sqrt{3} - i)$
14. If the equation $\cos^4 \theta + \sin^4 \theta + \lambda = 0$ has real solution for θ , then λ lies in the interval [JEE-2020]
 (A) $\left(-\frac{1}{2}, -\frac{1}{4}\right)$ (B) $\left(-1, -\frac{1}{2}\right)$
 (C) $\left(-\frac{3}{2}, -\frac{5}{4}\right)$ (D) $\left(-\frac{3}{2}, -1\right)$
15. If a ΔABC has vertices $A(-1, 7)$, $B(-7, 1)$ and $C(5, -5)$, then its orthocentre has coordinates [JEE-2020]
 (A) $(-3, 3)$ (B) $\left(-\frac{3}{5}, \frac{3}{5}\right)$
 (C) $\left(\frac{3}{5}, -\frac{3}{5}\right)$ (D) $(3, -3)$
16. If $L = \sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$ and $M = \cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$, then [JEE-2020]
 (A) $M = \frac{1}{2\sqrt{2}} + \frac{1}{2} \cos \frac{\pi}{8}$
 (B) $M = \frac{1}{4\sqrt{2}} + \frac{1}{4} \cos \frac{\pi}{8}$
 (C) $L = -\frac{1}{2\sqrt{2}} + \frac{1}{2} \cos \frac{\pi}{8}$
 (D) $L = \frac{1}{4\sqrt{2}} - \frac{1}{4} \cos \frac{\pi}{8}$
17. A ray of light coming from the point $(2, 2\sqrt{3})$ is incident at an angle 30° on the line $x = 1$ at the point A. The ray gets reflected on the line $x = 1$ and meets x -axis at the point B. Then, the line AB passes through the point [JEE-2020]
 (A) $(4, -\sqrt{3})$ (B) $\left(3, -\frac{1}{\sqrt{3}}\right)$
 (C) $(3, -\sqrt{3})$ (D) $\left(4, -\frac{\sqrt{3}}{2}\right)$
18. The angle of elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be 45° . After walking a distance of 80 meters towards the top, up a slope inclined at an angle of 30° to the horizontal plane, the angle of elevation of the top of the hill becomes 75° . Then the height of the hill (in meters) is _____. [JEE-2020]
19. The angle of elevation of the summit of a mountain from a point on the ground is 45° . After climbing up one km towards the summit at an inclination of 30° from the ground, the angle of elevation of the summit is found to be 60° . Then the height (in km) of the summit from the ground is [JEE-2020]
 (A) $\frac{1}{\sqrt{3} + 1}$ (B) $\frac{\sqrt{3} + 1}{\sqrt{3} - 1}$
 (C) $\frac{\sqrt{3} - 1}{\sqrt{3} + 1}$ (D) $\frac{1}{\sqrt{3} - 1}$

20. If $0 < \theta, \phi < \frac{\pi}{2}$, $x = \sum_{n=0}^{\infty} \cos^{2n}\theta$, $y = \sum_{n=0}^{\infty} \sin^{2n}\phi$ and

$$z = \sum_{n=0}^{\infty} \cos^{2n}\theta \cdot \sin^{2n}\phi \text{ then} \quad [\text{JEE 2021}]$$

- (A) $xy - z = (x + y)z$ (B) $xy + yz + zx = z$
 (C) $xyz = 4$ (D) $xy + z = (x + y)z$
21. A man is observing, from the top of a tower, a boat speeding towards the tower from a certain point A, with uniform speed. At that point, angle of depression of the boat with the man's eye is 30° (Ignore man's height). After sailing for 20 seconds, towards the base of the tower (which is at the level of water), the boat has reached a point B, where the angle of depression is 45° . Then the time taken (in seconds) by the boat from B to reach the base of the tower is [JEE 2021]
- (A) 10 (B) $10\sqrt{3}$
 (C) $10(\sqrt{3}+1)$ (D) $10(\sqrt{3}-1)$
22. If for $x \in \left(0, \frac{\pi}{2}\right)$, $\log_{10}\sin x + \log_{10}\cos x = -1$ and $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$, $n > 0$, then the value of n is equal to [JEE 2021]
- (A) 20 (B) 12
 (C) 9 (D) 16

23. If $15\sin^4\alpha + 10\cos^4\alpha = 6$, for some $\alpha \in \mathbb{R}$, then the value of $27\sec^6\alpha + 8\operatorname{cosec}^6\alpha$ is equal to [JEE 2021]
- (A) 350 (B) 500
 (C) 400 (D) 250

24. A spherical gas balloon of radius 16 meter subtends an angle 60° at the eye of the observer A while the angle of elevation of its center from the eyes of A is 75° . Then the height (in meter) of the top most point of the balloon from the level of the observer's eye is [JEE 2021]

(A) $8(\sqrt{2} + 2 + \sqrt{3})$ (B) $8(\sqrt{6} + \sqrt{2} + 2)$
 (C) $8(2 + 2\sqrt{3} + \sqrt{2})$ (D) $8(\sqrt{6} - \sqrt{2} + 2)$

25. Two poles, AB of length a metres and CD of length $a + b$ ($b \neq a$) metres are erected at the same horizontal level with bases at B and D. If $BD = x$ and $\tan \angle ACB = \frac{1}{2}$, then [JEE 2021]

(A) $x^2 + 2(a + 2b)x - b(a + b) = 0$
 (B) $x^2 + 2(a + 2b)x + a(a + b) = 0$
 (C) $x^2 - 2ax + b(a + b) = 0$
 (D) $x^2 - 2ax + a(a + b) = 0$

26. The number of solutions of the equation $|\cot x| = \cot x + \frac{1}{\sin x}$ in the interval $[0, 2\pi]$ is [JEE 2021]

JEE Advanced

Single Correct Option-type Questions

1. The number of integral values of k for which the equation $7 \cos x + 5 \sin x = 2k + 1$ has a solution is [JEE-2002]
- (A) 4 (B) 8
 (C) 10 (D) 12
2. $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = \frac{1}{e}$, where $\alpha, \beta \in [-\pi, \pi]$, number of pairs of α, β which satisfy both the equations is [JEE-2005]
- (A) 0 (B) 1
 (C) 2 (D) 4
3. If $0 < \theta < 2\pi$, then the intervals of values of θ for which $2 \sin^2\theta - 5 \sin \theta + 2 > 0$, is [JEE-2006]

(A) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$
 (B) $\left(\frac{\pi}{8}, \frac{5\pi}{6}\right)$
 (C) $\left(0, \frac{\pi}{8}\right) \cup \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$
 (D) $\left(\frac{41\pi}{48}, \pi\right)$

4. The number of solutions of the pair of equations $2 \sin^2\theta - \cos 2\theta = 0$ and $2 \cos^2\theta - 3 \sin \theta = 0$ in the interval $[0, 2\pi]$ is [JEE-2007]
- (A) zero (B) one
 (C) two (D) four

5. Let $P = \{\theta : \sin \theta - \cos \theta = \sqrt{2} \cos \theta\}$ and $Q = \{\theta : \sin \theta + \cos \theta = \sqrt{2} \sin \theta\}$ be two sets. Then [JEE-2011]

(A) $P \subset Q$ and $Q - P \neq \emptyset$
 (B) $Q \subset P$
 (C) $P \not\subset Q$
 (D) $P = Q$

6. For $x \in (0, \pi)$, the equation $\sin x + 2 \sin 2x - \sin 3x = 3$ has [JEE-2014]

(A) infinitely many solutions
 (B) three solutions
 (C) one solution
 (D) no solution

7. Let $S = \left\{x \in (-\pi, \pi) : x \neq 0, \pm \frac{\pi}{2}\right\}$, The sum of all distinct solutions of the equation $\sqrt{3} \sec x + \operatorname{cosec} x + 2 (\tan x - \cot x) = 0$ in the set S is equal to [JEE-2016]

(A) $-\frac{7\pi}{9}$ (B) $-\frac{2\pi}{9}$
 (C) 0 (D) $\frac{5\pi}{9}$

8. The value of $\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4} + \frac{(k-1)\pi}{6}\right) \sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}$ is [JEE-2016]

(A) $3 - \sqrt{3}$ (B) $2(3 - \sqrt{3})$
 (C) $2(\sqrt{3} - 1)$ (D) $2(2 + \sqrt{3})$

9. Let $\frac{\pi}{2} < x < \pi$ be such that $\cot x = \frac{-5}{\sqrt{11}}$. Then

$$\left(\sin \frac{11x}{2}\right)(\sin 6x - \cos 6x) + \left(\cos \frac{11x}{2}\right)(\sin 6x + \cos 6x)$$

is equal to: [JEE Advanced 2024]

(A) $\frac{\sqrt{11}-1}{2\sqrt{3}}$ (B) $\frac{\sqrt{11}+1}{2\sqrt{3}}$
 (C) $\frac{\sqrt{11}+1}{3\sqrt{2}}$ (D) $\frac{\sqrt{11}-1}{3\sqrt{2}}$

Multiple Correct Option-type Questions

1. Let α and β be an nonzero real numbers such that $2(\cos \beta - \cos \alpha) + \cos \alpha \cos \beta = 1$. Then which of the following is/are true? [JEE-2017]

(A) $\tan\left(\frac{\alpha}{2}\right) + \sqrt{3} \tan\left(\frac{\beta}{2}\right) = 0$

(B) $\sqrt{3} \tan\left(\frac{\alpha}{2}\right) + \tan\left(\frac{\beta}{2}\right) = 0$

(C) $\tan\left(\frac{\alpha}{2}\right) - \sqrt{3} \tan\left(\frac{\beta}{2}\right) = 0$

(D) $\sqrt{3} \tan\left(\frac{\alpha}{2}\right) - \tan\left(\frac{\beta}{2}\right) = 0$

(E) None of the above

2. Let x, y and z be positive real numbers. Suppose x, y and z are the lengths of the sides of a triangle opposite to its angles X, Y and Z , respectively. If

$$\tan \frac{x}{2} + \tan \frac{z}{2} = \frac{2y}{x+y+z},$$
 then which of the following

statements is/are TRUE? [JEE-2020]

(A) $2Y = X + Z$ (B) $Y = X + Z$

(C) $\tan \frac{x}{2} = \frac{x}{y+z}$ (D) $x^2 + z^2 - y^2 = xz$

Matrix Match-type Questions

1. Let $f(x) = \sin(\pi \cos x)$ and $g(x) = \cos(2\pi \sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in the increasing order

$$X = \{x : f(x) = 0\}, \quad Y = \{x : f'(x) = 0\},$$

$$Z = \{x : g(x) = 0\}, \quad W = \{x : g'(x) = 0\},$$

Column-I contains the sets X, Y, Z and W . Column-II contains some information regarding these sets. [JEE-2019]

Column I	Column II
(I) X	(P) $\supseteq \left\{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\right\}$
(II) Y	(Q) an arithmetic progression
(III) Z	(R) NOT an arithmetic progression
(IV) W	(S) $\supseteq \left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\right\}$
	(T) $\supseteq \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$
	(U) $\supseteq \left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$

Which of the following is the only CORRECT combination?

- (A) (III), (P), (Q), (U)
- (B) (IV), (P), (R), (S)
- (C) (III), (R), (U)
- (D) (IV), (Q), (T)

2. Consider the following lists:

Column I	Column II
(I) $\left\{x \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right] : \cos x + \sin x = 1\right\}$	(P) has two elements
(II) $\left\{x \in \left[-\frac{5\pi}{18}, \frac{5\pi}{18}\right] : \sqrt{3}\tan 3x = 1\right\}$	(Q) has three elements
(III) $\left\{x \in \left[-\frac{6\pi}{5}, \frac{6\pi}{5}\right] : 2\cos(2x) = \sqrt{3}\right\}$	(R) has four elements
(IV) $\left\{x \in \left[-\frac{7\pi}{4}, \frac{7\pi}{4}\right] : \sin x - \cos x = 1\right\}$	(S) has five elements
	(T) has six elements

The correct option is: [JEE Advanced 2022]

- (A) (I) → (P); (II) → (S); (III) → (P); (IV) → (S)
- (B) (I) → (P); (II) → (P); (III) → (T); (IV) → (R)
- (C) (I) → (Q); (II) → (P); (III) → (T); (IV) → (S)
- (D) (I) → (Q); (II) → (S); (III) → (P); (IV) → (R)

Integer-type Questions

- The number of values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for $n = 0, \pm 1, \pm 2$ and $\tan \theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is [JEE-2010]

- The number of all possible values of θ when $\theta \in (0, \pi)$ for which the system of equation [JEE-2010]

$$(y + z) \cos 3\theta = (xyz) \sin 3\theta$$

$$x \sin 3\theta = \frac{2 \cos 3\theta}{y} + \frac{2 \sin 3\theta}{z}$$

$(xyz) \sin 3\theta = (y + 2z)\cos 3\theta + y \sin 3\theta$ have a solution (x_0, y_0, z_0) with $y_0, z_0 \neq 0$ is

- Let a, b, c be three non-zero real numbers such that the equation $\sqrt{3}a \cos x + 2b \sin x = c, x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, has two distinct real roots α and β with $\alpha + \beta = \frac{\pi}{3}$. Then, the value of $\frac{b}{a}$ is

- The number of distinct solution of equation $\frac{5}{4} \cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$ in the interval $[0, 2\pi]$ is. [JEE-2015]

- Let $f: [0, 2] \rightarrow \mathbb{R}$ be the function defined by $f(x) = (3 - \sin(2\pi x)) \sin\left(\pi x - \frac{\pi}{4}\right) - \sin\left(3\pi x + \frac{\pi}{4}\right)$. If $\alpha, \beta \in [0, 2]$ are such that $\{x \in [0, 2] : f(x) \geq 0\} = [\alpha, \beta]$, then the value of $\beta - \alpha$ is [JEE-2020]



ANSWER KEYS

Check Your Understanding-I

1. 4 2. 7

3. Min. value = 3^{-5} for $x = (4n - 1)\frac{\pi}{4} - \frac{1}{2}\tan^{-1}\frac{3}{4}$, $n \in I$; Max. value = 3^5 for $x = (4n + 1)\frac{\pi}{4} - \tan^{-1}\frac{3}{4}$, $n \in I$

4. ϕ 5. $30^\circ, 150^\circ, 210^\circ, 330^\circ$

Check Your Understanding-II

1. $x = n\pi + (-1)^n \frac{\pi}{6}$ and $y = m\pi \pm \frac{\pi}{6}$ where m and n are integers.

2. $x = 2n\pi \pm \frac{2\pi}{3}$, $n \in I$

3. $\alpha - 2\pi$; $\alpha - \pi$, α , $\alpha + \pi$, where $\tan \alpha = \frac{2}{3}$ 4. $30^\circ, 45^\circ, 90^\circ, 135^\circ, 150^\circ$

5. $x = 2n\pi$ or $x = nx + (-1)^n \left(-\frac{\pi}{2}\right)$ or $x = nx + (-1)^n \frac{\pi}{6}$

6. $\frac{(2n+1)\pi}{4}$, $k\pi$, where $n, k \in I$

Practice Exercises

1. (a) $n\pi + (-1)^n \frac{\pi}{4}$, $n \in I$ (b) $n\pi + \frac{\pi}{3} + 1$, $n \in I$ (c) $n\pi - \frac{\pi}{4}$, $n \in I$ (d) $n\pi + (-1)^n \frac{\pi}{3}$, $n \in I$ (e) $n\pi \pm \frac{\pi}{4}$, $n \in I$

2. $\frac{n\pi}{4}$, $n \in I$ or $\frac{(2n+1)\pi}{10}$, $n \in I$ 3. $2n\pi \pm \frac{\pi}{3}$, $n \in I$

4. $\left(2n + \frac{1}{2}\right)\frac{\pi}{5}$, $n \in I$ or $2n\pi - \frac{\pi}{2}$, $n \in I$

5. $\left(n + \frac{1}{2}\right)\frac{\pi}{9}$, $n \in I$

6. $n\pi + \frac{\pi}{3}$, $n \in I$ or $n\pi + \frac{\pi}{4}$, $n \in I$

7. 45° and 60°

8. $n\pi + (-1)^n \frac{\pi}{10}$, $n \in I$ or $n\pi - (-1)^n \frac{3\pi}{10}$, $n \in I$

9. $\left(n + \frac{1}{4}\right)\frac{\pi}{2}$, $n \in I$

10. $\frac{n\pi}{3}$, $n \in I$ or $\left(n \pm \frac{1}{3}\right)\pi$, $n \in I$ 11. $2n\pi$, $n \in I$ or $\frac{2n\pi}{3} + \frac{\pi}{6}$, $n \in I$

12. $30^\circ, 45^\circ, 90^\circ, 135^\circ, 150^\circ$

13. $x = (2n + 1)\frac{\pi}{4}$, $n \in I$ or $x = (2n + 1)\frac{\pi}{2}$, $n \in I$ or $x = n\pi \pm \frac{\pi}{6}$, $n \in I$

14. $m\pi$, $m \in I$ or $\frac{m\pi}{n-1}$, $m \in I$ or $\left(m + \frac{1}{2}\right)\frac{\pi}{n}$, $m \in I$ 15. $n\pi + \frac{\pi}{6} + (-1)^n \frac{\pi}{4}$, $n \in I$

16. $2n\pi + \frac{2\pi}{3}$, $n \in I$

17. $2n\pi + \frac{\pi}{2}$, $n \in I$ or $2n\pi + 2\alpha$ where $\alpha = \tan^{-1}\frac{3}{7}$, $n \in I$

18. $n\pi \pm \frac{\pi}{6}$, $n \in I$

19. $\left(n + \frac{1}{3}\right)\frac{\pi}{3}$, $n \in I$

20. $x = \frac{n\pi}{3} - \frac{\pi}{9}$, $n \in I$

21. $\theta = n\pi + \frac{\pi}{4}$, $\phi = n\pi + (-1)^n \frac{\pi}{6}$, $n \in I$

22. $\theta = \frac{7\pi}{12}, \frac{19\pi}{12}$

23. $x = 2n\pi + \frac{\pi}{6}$, $n \in I$ 24. $0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}$ and π

25. $x = \frac{n\pi}{7} - \frac{\pi}{84}$ or $x = \frac{n\pi}{4} - \frac{5\pi}{48}, n \in I$ 26. $\frac{\pi}{8}, \frac{\pi}{3}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{2\pi}{3}, \frac{7\pi}{8}$
27. $A = 15^\circ, B = 30^\circ$ 28. $x = -1, y = n\pi \pm \frac{\pi}{4} + 1$
30. $n\pi, n\pi + (-1)^n \frac{\pi}{10}$ or $n\pi + (-1)^n \left(-\frac{3\pi}{10}\right)$ 31. $n\pi + \frac{\pi}{8} < x < n\pi + \frac{\pi}{4}$
32. $\frac{1}{2} [n\pi + (-1)^n \sin^{-1}(1 - \sqrt{2a+3})]$ where $n \in I$ and $a \in \left[-\frac{3}{2}, \frac{1}{2}\right]$ 33. $x = 2n\pi + \frac{3\pi}{4}, n \in I$
34. $\left\{\frac{\pi}{12}, \frac{5\pi}{12}\right\}$ 35. $x \in \left(2n\pi + \frac{\pi}{4}, 2n\pi + \frac{3\pi}{4}\right) \cup \left(2n\pi - \frac{\pi}{4}, 2n\pi\right) \cup \left((2n+1)\pi, 2n\pi + \frac{5\pi}{4}\right)$
36. $a = 0$ or $a < -1$ 37. $x = \frac{n\pi}{7} - \frac{\pi}{84}$ or $x = \frac{n\pi}{4} + \frac{7\pi}{48}, n \in I$ 38. $x = 2n\pi \pm \frac{2\pi}{3}, n \in I$
39. $\alpha - 2\pi; \alpha - \pi, \alpha, \alpha + \pi$, where $\tan \alpha = \frac{2}{3}$ 40. $x = 2nx + \frac{\pi}{12}$ or $2n\pi + \frac{17\pi}{12}; n \in I$

Assessment

JEE Main

Level I

- | | | | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1. B | 2. D | 3. D | 4. A | 5. B | 6. C | 7. B | 8. A | 9. B | 10. D |
| 11. A | 12. A | 13. D | 14. B | 15. A | 16. C | 17. A | 18. B | 19. B | 20. B |
| 21. A | 22. C | 23. C | 24. B | 25. A | 26. D | 27. D | 28. A | 29. B | 30. B |
| 31. C | 32. B | 33. A | 34. A | 35. B | | | | | |

Level II

- | | | | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1. B | 2. C | 3. B | 4. C | 5. A | 6. B | 7. D | 8. D | 9. C | 10. B |
| 11. C | 12. B | 13. C | 14. B | 15. D | 16. A | 17. C | 18. B | 19. A | 20. A |

Integer-type Questions

1. 0.52 2. 0.19 3. 4 4. 46 5. 3.5 6. 4 7. 564 8. 13 9. 8

JEE Advanced

Single Correct Option-type Questions

- | | | | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1. D | 2. C | 3. C | 4. D | 5. C | 6. D | 7. B | 8. B | 9. D | 10. B |
| 11. C | 12. A | 13. D | 14. A | 15. B | 16. D | 17. B | 18. B | 19. B | 20. C |
| 21. B | 22. A | 23. C | 24. A | 25. C | 26. C | 27. B | 28. B | | |

Multiple Correct Option-type Questions

- | | | | | | | |
|---------------|-------------|-------------|------------|-------------|-------------|----------------|
| 1. A, B, C, D | 2. A, D | 3. B, D | 4. A, B, C | 5. A, C | 6. B, C, D | 7. A, B, C, D |
| 8. A, B, C, D | 9. A, C | 10. C, D | 11. B, C | 12. B, C | 13. A, D | 14. A, B, C, D |
| 15. B, C, D | 16. A, B, C | 17. A, C, D | | 18. B, C, D | 19. A, C, D | 20. B, C |

Comprehension-based Questions

1. A 2. B 3. C 4. D 5. D 6. D 7. C 8. D 9. B 10. C
11. C

Matrix Match-type Questions

1. (A)–(P), (B)–(Q), (C)–(R), (D)–(S) 2. (A)–(Q), (B)–(R), (C)–(P), (D)–(R)
3. (A) P; (B) Q; (C) P; (D) T; (E) R 4. (A) P, R; (B) Q, S; (C) P, R 5. (A) R; (B) S; (C) P; (D) Q

Integer-type Questions

1. 2 2. 3 3. 0 4. 2 5. 3 6. 8 7. 2010 8. 10 9. 5050 10. 8
11. 4 12. 5 13. 10 14. 1 15. 4 16. 19 17. 40 18. 13 19. 4
20. 3.14 21. 1.11 22. 0.19

JEE Archives**JEE Main**

1. B 2. A 3. D 4. B 5. D 6. C 7. B 8. C 9. A 10. C
11. A 12. C 13. C 14. B 15. A 16. A 17. C 18. 80 19. D 20. D
21. C 22. B 23. D 24. B 25. C 26. 1

JEE Advanced**Single Correct Option-type Questions**

1. B 2. D 3. A 4. C 5. D 6. D 7. C 8. C 9. B

Multiple Correct Option-type Questions

1. E 2. B, C

Matrix Match-type Questions

1. B 2. B

Integer-type Questions

1. 3 2. 3 3. 0.5 4. 8 5. 1