NEET Class Companion Physics

Module 2

Contents

CHAPTER

7

Gravitation

Learning Objectives

During the course of this chapter, you will be able to:

- ☑ explain Kepler's laws.
- ☑ understand Universal law of gravitation.
- ☑ explain the gravitational constant.
- ☑ describe acceleration due to gravity of the earth.
- ☑ understand acceleration due to gravity below and above the surface of earth.
- ☑ describe gravitational potential energy.
- ☑ explain escape speed.
- ☑ understand earth satellites.
- ☑ explain energy of an orbiting satellite.
- ☑ differentiate geostationary and polar satellites.
- ☑ understand weightlessness.

INTRODUCTION

Early in our lives, we become aware of the tendency of all material objects to be attracted towards the earth. Anything thrown up falls down towards the earth, going uphill is lot more tiring than going downhill, raindrops from the clouds above fall towards the earth and there are many other such phenomena. Historically it was the Italian Physicist Galileo (1564-1642) who recognised the fact that all bodies, irrespective of their masses, are accelerated towards the earth with a constant acceleration.

KEPLER'S LAWS

First Law (Law of Orbits)

Every planet revolves around the sun in an elliptical orbit and sun is at one focus.

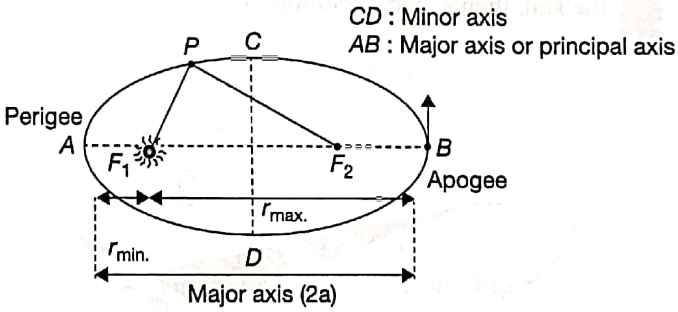


Figure 7.1 Kepler's first law

Ellipse: Ellipse is a set of all points in a plane such that sum of distances from two fixed points is a constant. These two fixed points are known as the foci of the ellipse.

According to figure,

$$PF_1 + PF_2 = AF_1 + AF_2 = BF_1 + BF_2 = \text{constant}$$

But in ellipse, $AF_1 = BF_2$ (minimum distance from both focus is same)

$$PF_1 + PF_2 = BF_2 + AF_2 = BF_1 + AF_1 = 2a$$

$$r_1 + r_2 = r_{min} + r_{max} = 2a$$

$$\therefore\ a = \frac{r_1 + r_2}{2} = \frac{r_{min} + r_{max}}{2} = \text{(Mean distance)}$$

Second Law (Law of Areas)

According to this law, the radius vector joining sun and planet sweeps out equal area in equal interval of time.

If $t_1 = t_2$, then $A_1 = A_2$

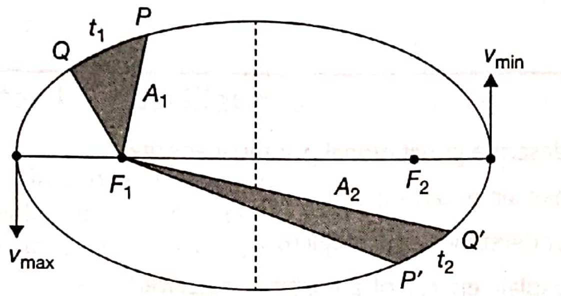


Figure 7.2 Kepler's second law

> **NOTES**
>
> When a planet reaches nearest to the sun, then speed of planet is maximum and when it is farthest from the sun, then speed is minimum.

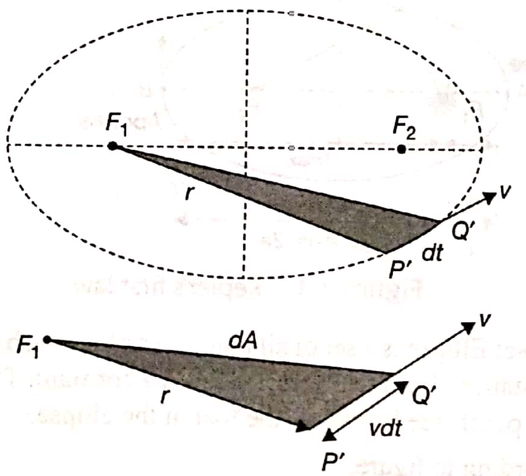


Figure 7.3 Area covered by planet in short interval

Area covered by planet in short interval of time is,

$$dA = \frac{1}{2} \times \text{Base} \times \text{Height} \Rightarrow dA = \frac{1}{2} \times r \times vdt$$

$\frac{dA}{dt} = \frac{1}{2} rv$ or areal speed of planet is always constant.

$$\frac{dA}{dt} = \frac{1}{2} rv = \text{Constant}$$

Important Points

1. We know that in planetary motion

$$\frac{1}{2} rv = \text{Constant} \Rightarrow \frac{1}{2} rv \times \frac{m}{m} = \text{Constant}$$

$$\frac{mvr}{2m} = \text{Constant} \Rightarrow mvr = \text{Constant}$$

$$\Rightarrow L = \text{Constant}$$

So in planetary motion, angular momentum of the planet is always conserved (COAM).

2. Apply COAM between points (A) and (B),

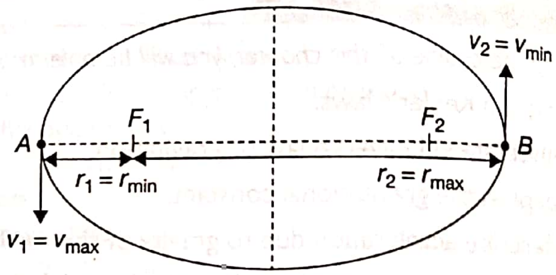


Figure 7.4

$$L_A = L_B$$

$$mv_{max}\, r_{min} = m \cdot v_{min}\, r_{max}$$

$$v_{max} r_{min} = v_{min} r_{max}$$

A planet moving around the sun in an elliptical path having eccentricity e and semi major axis a is shown as follows.

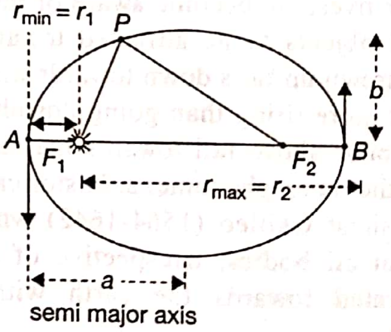


Figure 7.5 Representation of a planet moving around the sun in an elliptical path

For an ellipse: Its general equation is given as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

If $a > b$, then a is semi-major axis, b is semi-minor axis and e is eccentricity where,

$$b^2 = a^2(1 - e^2) \Rightarrow e = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$

Apply the conservation of angular momentum (COAM) at the aphelion and perihelion to get

$$mv_p r_p = mv_a r_a$$

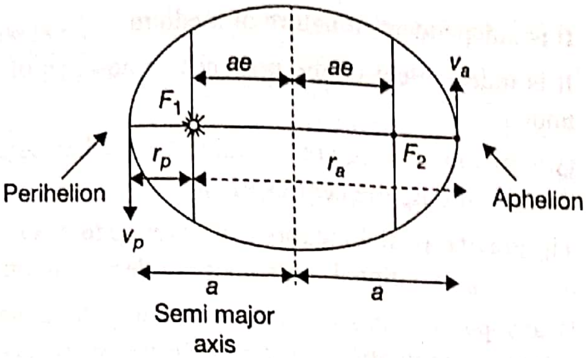


Figure 7.6

$$r_{max} = a(1+e);\ r_{min} = a(1-e)$$

$$\Rightarrow \frac{v_p}{v_a} = \frac{v_{max}}{v_{min}} = \frac{1+e}{1-e} \quad (1)$$

By conservation of mechanical energy,

$$\frac{1}{2}mv_p^2 - \frac{GMm}{r_p} = \frac{1}{2}mv_a^2 - \frac{GMm}{r_a} = -\frac{GMm}{2a} \quad (2)$$

By solving Eqs. (1) and (2),

$$\Rightarrow v_a = \sqrt{\frac{GM}{a}\left(\frac{1-e}{1+e}\right)};\ v_p = \sqrt{\frac{GM}{a}\left(\frac{1+e}{1-e}\right)}$$

Third Law—Time Period Law (Law of Periods)

Square of the time period is directly proportional to the cube of the mean distance (semi-major axis).

$$T^2 \propto r^3$$

$$T^2 = K_s r^3 \quad (1)$$

where K_s = Kepler's constant

$$K_s = \frac{4\pi^2}{GM_s} = 2.97 \times 10^{-17}\ \text{s}^2/\text{m}^3\ (M_s = \text{Mass of Sun})$$

Substituting the value of K_s in Eq. (1),

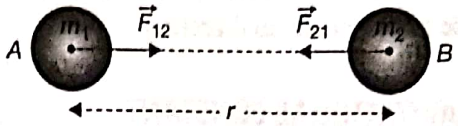
$$T^2 = \frac{4\pi^2}{GM_s} r^3$$

Key Points:

1. Time period of earth on its own axis is $T_e = 24$ hours.
2. Time period of earth about sun is $T = 365$ days.
3. The period of moon around earth is $T_m = 27.3$ days which is also called **Lunar month**.
4. Time period of Geostationary satellite is 24 hours.

UNIVERSAL LAW OF GRAVITATION

According to the Newton's law of gravitation, there is a force of attraction applied by a body on every other body of the universe. This force is directly proportional to the product of masses of two bodies and inversely proportional to the square of the distance between the bodies.



According to Newton's law

$$F \propto m_1 m_2 \quad (1)$$

$$F \propto \frac{1}{r^2} \quad (2)$$

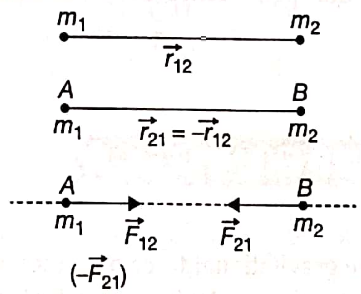
From Eqs. (1) and (2), we get

$$F \propto \frac{m_1 m_2}{r^2}$$

$$\therefore\ F = -\frac{Gm_1m_2}{r^2}$$

where G = Universal gravitational constant

Vector Form of Newton's Law of Gravitation



Let

$\vec{r}_{12}$ = Displacement vector from A to B

$\vec{r}_{21}$ = Displacement vector from B to A

$\vec{F}_{21}$ = Gravitational force exerted on B by A

$\vec{F}_{12}$ = Gravitational force exerted on A by B

Gravitational force,

$$\vec{F}_{12} = -\frac{Gm_1m_2}{r_{21}^2}\hat{r}_{21} = -\frac{Gm_1m_2}{r_{21}^3}\vec{r}_{21}$$

$$= +\frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12} = +\frac{Gm_1m_2}{r_{12}^3}\vec{r}_{12}$$

Negative sign shows that:

1. The direction of $\vec{F}_{12}$ is opposite to that $\hat{r}_{21}$.
2. The gravitational force is attractive in nature.

Similarly,

$$\vec{F}_{21} = -\frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12}$$

Conclusion: $\vec{F}_{12} = -\vec{F}_{21}$

The gravitational force between two bodies are equal in magnitude and opposite in direction.

THE GRAVITATIONAL CONSTANT

1. Gravitational constant is a scalar quantity.
2. Its value is $G = 6.67 \times 10^{-11}$ N-m²/kg² (SI unit) and CGS value is 6.67×10^{-8} dyne-cm²/g².
3. Dimension of gravitational constant is $[M^{-1}L^3T^{-2}]$.
4. Its value is same throughout the universe and is independent of the nature and size of the bodies and nature of the medium between the bodies.
5. Its values were first found out by the 'Torsion Balance' by scientist 'Cavendish'.

Important Points about Gravitational Force

1. Gravitational forces are developed in form of action and reaction pair. Hence obey Newton's third law of motion.
2. It is independent of nature of medium.
3. It is independent of the presence or absence of other bodies.
4. Gravitational forces are central forces as they act along the line joining the centres of two bodies.
5. The gravitational forces are conservative forces so work done by gravitational force does not depend upon path.
6. If any particle moves along circular path under the action of gravitational force, then the work done by this force is always zero.
7. It holds well over wide range and short range but it fails when the distance between the objects is less than 10^{-9} m (i.e., of the order of intermolecular distances).
8. Gravitational force is very weak.
9. Force developed between any two masses is called gravitational force and force between earth and any-body is called gravity.
10. The total gravitational force on one particle due to number of particles is the resultant of forces of attraction exerted on the given particle due to individual particles, i.e., $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 \ldots$ it means the principle of superimposition is valid.

SOLVED EXAMPLES – I

EXAMPLE 1

Calculate the gravitational force of attraction between two spherical balls of mass 10 kg each which are placed at a distance of 100 m from each other.

SOLUTION

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67\times10^{-11}\times10\times10}{(100)^2} = 6.67 \times 10^{-13}\text{ N}$$

EXAMPLE 2

Two particles having masses 1 kg and 2 kg are placed 50 cm apart. Calculate the initial acceleration of the massive particle assuming that they are affected by their mutual gravitation only.

SOLUTION

The force of gravitation exerted by one particle on another is,

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67\times10^{-11}\times1\times2}{(0.5)^2} = 5.3 \times 10^{-10}\text{ N}$$

Acceleration of heavier particle $= \frac{F}{m_2} = \frac{5.3\times10^{-10}}{2}$

$= 2.65 \times 10^{-10}$ m/s²

EXAMPLE 3

The value of universal gravitational constant G depends upon:

(A) Nature of material of two bodies
(B) Medium between the two bodies
(C) Acceleration of two bodies
(D) None of these

SOLUTION

Because the Gravitational constant G is a universal constant. So answer is (D).

EXAMPLE 4

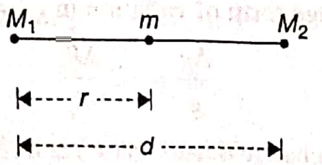
Two particles having masses M_1 and M_2 placed at d distance apart are at rest. A third particle present on the line joining the stationary particles experiences zero gravitational force. Find the distance of the third particle from the mass M_1.

SOLUTION

The force on m towards M_1 is,

$$F_1 = \frac{GM_1 m}{r^2}$$

The force on m towards M_2 is,



$$F_2 = \frac{GM_2 m}{(d-r)^2}$$

According to question net force on m is zero,

$$\therefore F_1 = F_2$$

$$\Rightarrow \frac{GM_1 m}{r^2} = \frac{GM_2 m}{(d-r)^2} \Rightarrow \left(\frac{d-r}{r}\right)^2 = \frac{M_2}{M_1}$$

$$\Rightarrow \frac{d}{r} - 1 = \frac{\sqrt{M_2}}{\sqrt{M_1}} \Rightarrow r = d\left(\frac{\sqrt{M_1}}{\sqrt{M_1}+\sqrt{M_2}}\right)$$

EXAMPLE 5

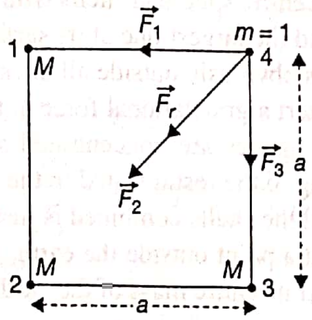
Three masses, M each are at three corners of a square having side a. What is the force experienced by a unit mass placed at the remaining corner of the square?

SOLUTION

$$F_1 = F_3 = \frac{GM}{a^2}$$

Resultant of F_1 and F_3

$$F_r = \sqrt{2}\frac{GM}{a^2}$$



And $$F_2 = \frac{GM}{(\sqrt{2}a)^2} = \frac{GM}{2a^2}$$

Now F_r and F_2 act in the same direction so net force on unit mass,

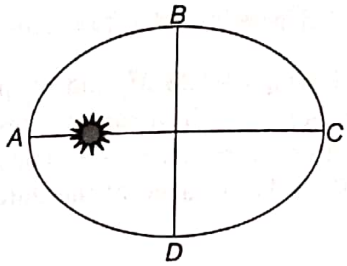
$$F_{net} = \frac{\sqrt{2}GM}{a^2} + \frac{GM}{2a^2} = \frac{GM}{a^2}\left[\sqrt{2}+\frac{1}{2}\right]$$ and it is directed along the diagonal passing through 4th corner.

CHECK YOUR UNDERSTANDING - I

1. Two particle of equal mass (m) move in a circle of radius (r) under the action of their mutual gravitational attraction. Find the speed of each particle.
2. Three equal particles each of mass (m) are placed at the three corners of an equilateral triangle of side (a). Find the force exerted by this system on another particle of mass (m) placed at:
   (a) The mid-point of a side.
   (b) At the centre of the triangle
3. A physical balance is balanced by 10 kg mass. If 1000 kg mass is placed at 1m below the one pan of the balance, find how much mass is placed on other pan so that physical balance is balanced.
4. A mass (M) is split into two parts (m) and ($M - m$). Which are then separated by a certain distance. What ratio of (m/M) will maximize the gravitational force between the parts?

5. Kepler's second law is a consequence of
   (A) conservation of kinetic energy
   (B) conservation of linear momentum
   (C) conservation of angular momentum
   (D) conservation of speed

6. In adjoining figure earth goes around the sun in elliptical orbit on which point the orbital speed is maximum:



(A) On A
(B) On B
(C) On C
(D) On D

ACCELERATION DUE TO GRAVITY OF THE EARTH

The earth can be imagined to be a sphere made up of a large number of concentric spherical shells with the smallest one at the centre and the largest one at its surface. A point outside the earth is obviously outside all the shells. Therefore, all the shells exert a gravitational force at the point outside just as if their masses are concentrated at their common centre according to the result stated in the last section. The total mass of all the shells combined is just the mass of the earth. Hence, at a point outside the earth, the gravitational force is just as if its entire mass of the earth is concentrated at its center.

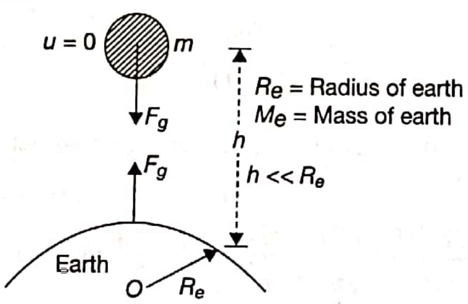


Figure 7.7

$$F_g = ma \Rightarrow \frac{GM_e m}{R_e^2} = m \cdot a_g$$

$$a_g = g = \frac{GM_e}{R_e^2}$$

$$GM_e = gR_e^2$$

Important Points

1. In form of density, $g = \frac{GM_e}{R_e^2} = \frac{G}{R_e^2} \times \frac{4}{3}\pi R_e^3 \times \rho$

$$\therefore\ g = \frac{4}{3}\pi G.R_e \rho$$

If ρ is constant, then $g \propto R_e$.

2. If M is constant,

$$g \propto \frac{1}{R^2}$$

Percentage form of variation in g (upto 5%),

$$\frac{\Delta g}{g} = -2.\frac{\Delta R_e}{R_e}$$

3. If small change occurs in (M) and (R), then

$$\frac{\Delta g}{g} = \frac{\Delta M}{M} - 2.\frac{\Delta R_e}{R_e}$$

If R and M are constant, $\frac{\Delta g}{g} = -2$

ACCELERATION DUE TO GRAVITY BELOW AND ABOVE THE SURFACE OF EARTH

Due to Altitude (Height)

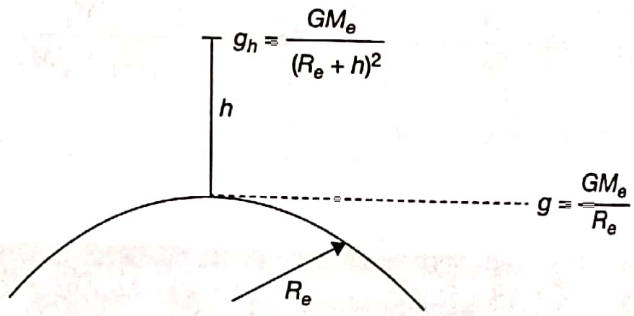


Figure 7.8

$$\frac{g_h}{g} = \frac{R_e^2}{(R_e + h)^2} = \frac{R_e^2}{R_e^2[1 + h/R_e]^2}$$

$$g_h = g\left[1 + \frac{h}{R_e}\right]^{-2}$$

Expand by binomial theorem,

$(1 + x)^n = 1 + n \cdot x + \frac{n \cdot (n-1)}{2!} . x^2 + \ldots$ if $x << 1$ higher power terms are negligible and $(1 + x)^n = 1 + nx \ldots$

$\because h << R_e$, higher power terms are negligible,

$$\therefore \left(1+\frac{h}{R_e}\right)^{-2} = \left(1-\frac{2h}{R_e}\right)$$

$$\therefore g_h = g\left[1-\frac{2h}{R_e}\right]$$

NOTES

This formula is valid if h is upto 5% of earth radius (320 km from earth surface).

If h is greater than 5% of earth radius we use,

$$g_h = \frac{GM}{(R_e+h)^2}$$

Due to Depth

Density of earth remains same throughout.

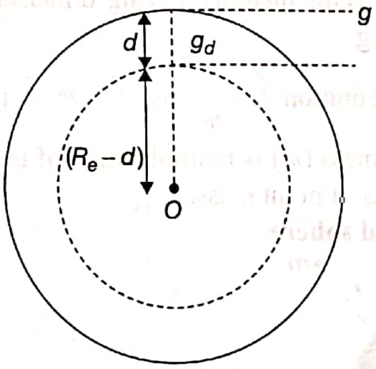


Figure 7.9

At earth surface:

$$g = \frac{4}{3}\pi G \cdot R_e \cdot \rho \quad (1)$$

At depth *d* inside the earth:

$$g_d = \frac{4}{3}\pi G(R_e-d)\rho \quad (2)$$

From Eqs. (1) and (2), we get

$$\frac{g_d}{g} = \frac{R_e-d}{R_e} = \left[1-\frac{d}{R_e}\right]$$

$$\therefore g_d = g\left[1-\frac{d}{R_e}\right]$$ valid for 100% depth.

Special Case:

$\therefore \Delta g_d = g - g_d =$ decrement in depth with depth

$$= g - g\left[1-\frac{d}{R_e}\right]$$

$$\Delta g_d = \frac{gd}{R_e} \qquad \therefore \frac{\Delta g_d}{g} = \frac{d}{R_e}$$

Due to shape of the Earth

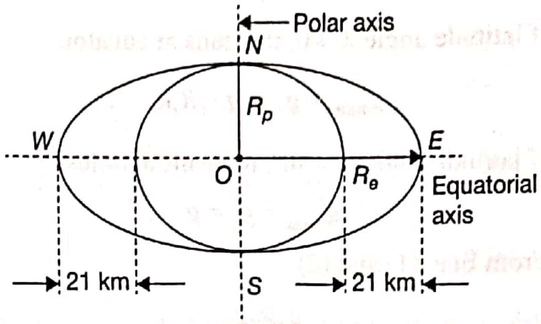


Figure 7.10

From the figure,

$$R_p < R_e$$

$$R_e = R_p + 21 \text{ km.}$$

$$g_p = \frac{GM_e}{R_p^2} \text{ and } g_e = \frac{GM_e}{(R_p+21)^2}$$

$$\therefore g_e < g_p$$

$$\therefore g_p - g_e = 0.02 \text{ m/s}^2$$

Due to Rotation of the Earth

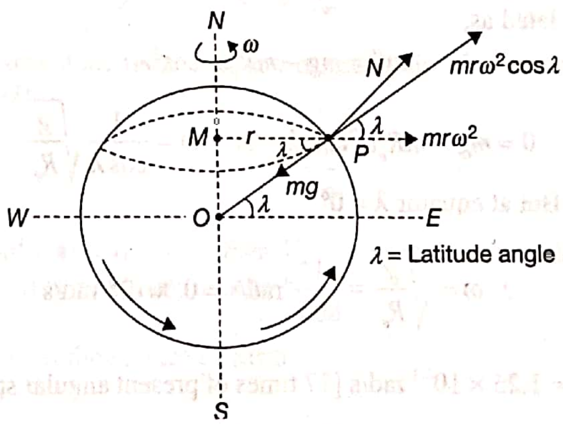


Figure 7.11

Net force on particle, $mg' = mg - mr\omega^2 \cos\lambda$

or $$g' = g - r\omega^2 \cos\lambda \quad (1)$$

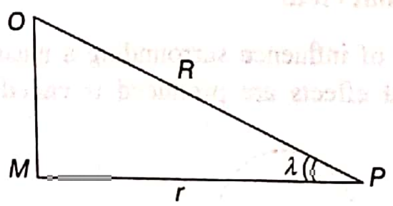


Figure 7.12

From ΔOMP

$$r = R_e \cos\lambda$$

Important Points

1. Gravitational potential is a scalar quantity and its unit and dimensions are J/kg. and $[L^2T^{-2}]$
2. As by definition of work, $W = \int \vec{F} \cdot d\vec{r}$

$$V = -\frac{W}{m} = -\int \frac{\vec{F} \cdot d\vec{r}}{m} \left[\text{As } \frac{\vec{F}}{m} = \vec{I} \right]$$

$$\therefore\ I = -\frac{dV}{dr} = -\text{ve potential gradient}$$

3. **For Solid Sphere**

Case 1: $r > R$ (outside the sphere)

$$V_{\text{outside}} = -\frac{GM}{r}$$

Case 2: $r = R$

$$V_{\text{surface}} = -\frac{GM}{R}$$

Case 3: $r < R$ (inside the sphere)

$$V_{\text{inside}} = -\frac{GM}{2R^3}[3R^2 - r^2]$$

It is clear that the potential ($|V|$) will be maximum at the centre ($r = 0$)

$$|V| = \frac{3}{2}\frac{GM}{R}$$

$$V_{\text{centre}} = \frac{3}{2}V_{\text{surface}}$$

For Hollow Sphere

Case 1: $r > R$ (outside the sphere)

$$V_{\text{outside}} = -\frac{GM}{r}$$

Case 2: $r = R$ (on the surface)

$$V_{\text{surface}} = -\frac{GM}{R}$$

Case 3: $r < R$ (Inside the sphere)

Potential everywhere is same as its value at the surface and equal to,

$$V_{\text{inside}} = -\frac{GM}{R}$$

SOLVED EXAMPLES – II

EXAMPLE 1

Particles having mass M each are placed along x–axis at positions $x = 1$ m, $x = 2$ m, $x = 4$ m and so on to infinity. Calculate the gravitation field intensity at the origin.

SOLUTION

$$\vec{I}_{\text{net}} = \vec{I}_1 + \vec{I}_2 + \vec{I}_3 + \vec{I}_4 + \cdots \infty \text{ terms}$$

$$= \frac{GM}{(1)^2}\hat{i} + \frac{GM}{(2)^2}\hat{i} + \frac{GM\hat{i}}{(4)^2} + \cdots \infty \text{ terms}$$

$$= GM\hat{i}\left(1 + \frac{1}{4} + \frac{1}{16} + \cdots \infty\right)$$

$$\left[\text{Here in the GP, } a = 1 \text{ and } r = \frac{1}{4}\right]$$

$$\text{So, } \vec{I}_{\text{net}} = GM\hat{i}\left[\frac{1}{1 - \frac{1}{4}}\right] = GM\hat{i}\left[\frac{1}{\left(\frac{3}{4}\right)}\right]$$

$$\vec{I}_{\text{net}} = \frac{4}{3}GM\hat{i}$$

EXAMPLE 2

Find the depth below the earth's surface where the acceleration due to gravity is decreased by 1%.

SOLUTION

$$\frac{\Delta g_d}{g} = \frac{d}{R_e} \Rightarrow \frac{1}{100} = \frac{d}{6400}$$

$$\therefore d = 64 \text{ km}$$

EXAMPLE 3

State whether the statements are true or false.

(A) If $r > R$, g decreases as we move away from the centre of the Earth.

(B) If $r < R$, g decreases as we move away from the centre of the Earth.

(C) At the centre of the Earth, g is zero.

(D) If earth stops rotation, value of g decreases.

SOLUTION

Variation of g distance: If $r > R$, then $g \propto \frac{1}{r^2}$

$\therefore$ (A) is correct

If $r < R$, then $g \propto r$.

$\therefore$ (B) is incorrect and (C) is correct.

Variation of g with ω: $g' = g - \omega^2 R \cos^2 \lambda$

If $\omega = 0$, then g will not change at poles where $\cos \lambda = 0$, while at other points g increases.

$\therefore$ (D) is incorrect.

EXAMPLE 4

What is the speed of rotation of Earth about its axis such that the person on the equator weighs $\frac{3}{5}$ of its present value? Express the answer in terms of R and g.

SOLUTION

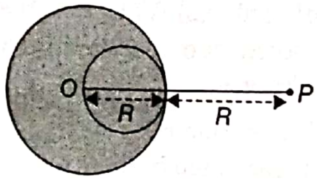
Weight on the equator $W' = \frac{3}{5}W \Rightarrow \frac{3}{5}mg = mg - m\omega^2 R$

$$\Rightarrow \omega = \sqrt{\frac{2g}{5R}}$$

EXAMPLE 5

A solid sphere having radius R and uniform density exerts a force F_1 on a particle P, which is at a distance $2R$ from the centre of the sphere. Then a spherical cavity having radius $R/2$ is formed in the sphere as shown in figure.

The modified sphere having a cavity applies a force F_2 on the same particle P. F_1 and F_2 are gravitational forces. Calculate the ratio F_2/F_1.



SOLUTION

$$F_1 = \frac{GMm}{4R^2}$$

F_2 = Force due to whole sphere - Force due to the sphere forming the cavity

$$F_2 = \frac{GMm}{4R^2} - \frac{GMm}{18R^2} \Rightarrow \frac{7GMm}{36R^2}$$

$$\therefore \frac{F_2}{F_1} = \frac{7}{9}$$

EXAMPLE 6

Estimate the maximum vertical distance through which an astronaut can jump on the moon. The corresponding distance on the earth is 0.5 m.

SOLUTION

$$\because mgh = \text{Constant}$$

$$\therefore h \propto \frac{1}{g} \Rightarrow h_m = \frac{h_e g_e}{g_m} = \frac{0.5 \times g}{g/6} = 3 \text{ m.}$$

CHECK YOUR UNDERSTANDING - II

1. In a certain region of space gravitational field is given by, $I = -(K/r)$. Taking reference point to be at $r = r_0$ with $V = V_0$. Find potential.
2. Two masses of 10^2 kg and 10^3 kg are separated by distance 1 m. Find gravitational potential at mid-point of the line joining them.
3. If radius of earth shrinks by 1%, then what will be change in acceleration due to gravity? (Mass of the earth is constant.)
4. At which height from earth surface, acceleration due to gravity is decreased by 1%?
5. Find the percentage decrease in the weight of the body when taken to a height of 16 km above the surface of earth. (radius of earth is 6400 km)
6. What is the value of acceleration due to gravity at height equal to half the radius of earth from surface of earth? (Take $g = 10$ m/s² at earth surface)
7. At which height from the earth surface acceleration due to gravity will be 1/4 of its value at earth's surface?
8. At which depth from earth surface, acceleration due to gravity is decreased by 1%?
9. At which height above earth's surface value of g is same as in a mine 100 km deep?
10. How much below the surface does the acceleration due to gravity becomes 70% of its value on the surface of earth?
11. At which depth from earth surface does the acceleration due to gravity becomes 1/4 times of g?

GRAVITATIONAL POTENTIAL ENERGY

The work done to bring a particle from infinity to some point A without changing its kinetic energy is the gravitational energy of the particle at point A.

$$W = U = -\frac{GMm}{r} \quad \text{or} \quad U = -\frac{Gm_1m_2}{r}$$

(Here, negative sign shows the boundness of the two bodies.)

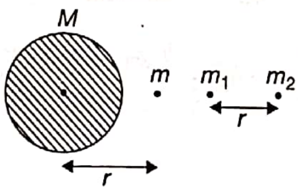
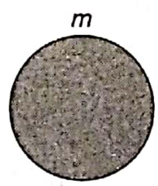


Figure 7.13

1. Gravitational energy is a scalar quantity.
2. Its dimensions are $[M^1L^2T^{-2}]$ and SI unit is Joules.
3. The gravitational energy of a particle having mass m placed on the Earth's surface is given as follows, the mass of earth is M and radius is R.

$$U = -\frac{GMm}{R}$$



Gravitational Potential Energy for Three Particles Systems

The net gravitational potential energy for a system is the sum of gravitational potential energy for all the possible pairs in the system.

For example,

$$U_{\text{system}} = \left(-\frac{Gm_1m_2}{r_1}\right) + \left(-\frac{Gm_2m_3}{r_2}\right) + \left(-\frac{Gm_1m_3}{r_3}\right)$$

$$U_{\text{system}} = -\frac{Gm_1m_2}{r_1} - \frac{Gm_2m_3}{r_2} - \frac{Gm_1m_3}{r_3}$$

The work done to raise a particle above the Earth's surface:

$$W = \Delta U = U_f - U_i$$

$$\Rightarrow W = -\frac{GMm}{R+h} - \left(-\frac{GMm}{R}\right)$$

$$\Rightarrow W = GMm\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$\Rightarrow W = GMm\left(\frac{R+h-R}{R(R+h)}\right)$$

$$\Rightarrow W = gR^2m\left(\frac{h}{R^2\left(1+\frac{h}{R}\right)}\right)$$

$[\because GM = gR^2]$

$$W = \frac{mgh}{\left(1+\frac{h}{R}\right)}$$

Special cases:

1. If $h << R$, then $\frac{h}{R} = 0$

$$\therefore W = \frac{mgh}{1+0} = mgh$$

2. If $h = R$,

then $W = \frac{mgR}{\left(1+\frac{R}{R}\right)} = \frac{mgR}{2}$.

The required velocity to project a particle from the Earth's surface to a height *h*:

Applying law of conservation of energy on the surface and at a height h.

$$(K+U)_{\text{surface}} = (K+U)_{\text{final}}$$

$$\Rightarrow \frac{1}{2}mv^2 - \frac{GMm}{R} = 0 - \frac{GMm}{R+h}$$

$$\Rightarrow \frac{1}{2}mv^2 - \frac{GMm}{R} = -\left[\frac{GMm}{R+h}\right]$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{mgh}{1+\frac{h}{R}}$$

$$\Rightarrow v^2 = \frac{2gh}{1+\frac{h}{R}}$$

$$\Rightarrow v = \sqrt{\frac{2gh}{1+\frac{h}{R}}}$$

NOTES

If a particle is released from height h from the Earth's surface, then its velocity on hitting the Earth's surface is

$$v = \sqrt{\frac{2gh}{1+\frac{h}{R}}}$$

The maximum height obtained by a particle projected from the Earth's surface with velocity v:

$$v^2 = \frac{2gh}{1+\frac{h}{R}}$$

$$\Rightarrow v^2 + \frac{v^2h}{R} = 2gh$$

$$\Rightarrow v^2 = 2gh - \frac{v^2h}{R}$$

$$\Rightarrow v^2 = h\left(2g - \frac{v^2}{R}\right)$$

$$\Rightarrow h = \frac{v^2}{2g - \frac{v^2}{R}} = \frac{v^2R}{2gR - v^2}$$

$$h = \frac{v^2R}{2gR - v^2}$$

ESCAPE SPEED (V_e)

The minimum speed required such that an object on the planet's surface just escapes the planet's gravitational field.